Analog Feedback Communication System with Receive Diversity and MRC

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Abstract – *This study investigates the analog feedback communication system (AFCS) architecture considering Rayleigh fading channel model and receive diversity maximum ratio combining (MRC). This architecture employs a power-efficient transmitting unit, an estimator at the receiver side and iterative algorithm that minimizes the mean square error (MSE). Using the feedback channel, the estimated received sample is fed back to the transmitter. The performance of AFCS with the Rayleigh fading channel model is evaluated using MSE optimization. The investigation revealed that when compared to a single input single output AFCS system, the diversity-enabled AFCS system achieves negligible MSE in fewer iterations. MSE of order 10^{-3} is achieved by 6 receive antennas with MRC in only 4 iterations at 0dB channel signal-to-noise ratio (SNR), compared to a single input single output AFCS system that requires more than 10 iterations to achieve the same order of MSE.*

Keywords: *Shannon Capacity, Analog Feedback Communication System, Receive Diversity, Maximum Ratio Combining, Optimum Performance Theoretically attainable*

1. INTRODUCTION

A prevailing trend in wireless communication is increasing data rates, reliable and long-range transmission while reducing transmitter sizes, energy usage, and the costs [1]. This task is internally incoherent since using low-power transmitters to save energy consumption results in decreased quality, range, and transmission rate. Many foundational works in information theory and communication theory depict the highest possible performance limits for the transmission [2], [3]. One such milestone work by Shannon showed that transmission of information at the highest channel capacity is possible in the presence of noise [4]. However, no strict analytical guidelines were provided for the actual implementation of such systems which work on the performance bounds. With the invention of turbo codes and sub- sequent developments [5, 6], some information theory promises have more or less become a reality. However, the most popular source and channel coding schemes do not offer performance even close to ideal when the block lengths are short. With large block lengths, performance is almost ideal but the cost is paid in terms of unconstrained power, delay and complexity.

Another problem with the separation theorem based digital systems is that whenever we want to change the code rate or the distortion target, we must completely redesign the digital system. Furthermore, digital systems are subject to the cliff effect [7]. The system's performance suffers greatly if the channel distortions are less than the designed values, while it improves only slightly if the channel condition improves.

Analog communications, which rely on the transmission of discrete-time continuous-amplitude sources can be viewed as a viable alternative to digital systems. Analog communication is well known to perform best under certain conditions, such as direct transmission of Gaussian samples over AWGN channels with absolutely no coding required [8]. P. Elias demonstrated [9] in the middle of the 1950s that if the information source is statistically matched to the channel, the analog communication systems can convey signals at the information limits without complex coding if a feedback channel is available.

Feedback does not improve the capacity of a memory less channel [3], but is definitely known to reduce error exponent and the coding complexity needed for achieving the performance limits [10]. This finding sparked intense curiosity among researchers and led to many significant works like [11]. The outcomes of these studies unequivocally showed that it is possible to design a perfect Analog Feedback Communication System (AFCS) whose performance reaches the limits of information and the complexity of AFCS systems is very less compared to the performance achieving digital systems. The peculiarity of these studies is that it directly optimizes the transmitter and receiver for a particular channel model, unlike current digital communication systems (DCS) where the source coder and channel coder optimization is done separately. However, towards the beginning of the 1970s, research in this area was reduced to almost nil and DCS became popular because of several advantages including ease of implementation, cost-effectiveness, and design. However, the research interest in AFCS has increased again, particularly for some applications like power efficient wireless sensor networks (WSN) [12, 13], small satellites [14, 15], radio frequency identification (RFID) modules, etc. AFCS architecture with a transmission and reception algorithm, an adaptive modulator, an estimator and a feedback channel are proposed and discussed in depth in works [16] [17]. Here MSE is emphasized to be the performance criteria and the relation of the optimality of the algorithm to the number of iterations is derived and presented. The research [18] and [19], demonstrated the possibilities and benefits of using feedback communication systems (FCS) as an estimating system. They discussed FCS optimization that eliminates the issues that led to the discontinuation of AFCS research and the solutions to those issues which allow for the design of the most efficient systems working at Shannon's bounds. Based on AFCS optimization, power-bandwidth efficiency and bit-rate expressions are discussed in [20]. These studies re-emphasized that the transmission over the "threshold" number of cycles n offers the "perfect" P-B trade-off and full utilization of the system's resources. Apart from the works mentioned so far, authors have discussed many resource-constrained use cases where AFCS might prove better compared to DCS [13], [14].

In All the research works carried out in the field of AFCS like [16] so far assumed the channel model to be additive white Gaussian (AWG). The real world wireless channel can be modelled better with Rayleigh or Rician fading channel model. The fading channel model and the techniques like diversity and multiple input multiple outputs (MIMO) to combat fading channel noise still remain unearthed and open for research. Hence, in this work we present the architecture of AFCS for wireless flat fading channels as a follow-up AFCS works. This is very important to maintain the research in this area and to come up with some practically implementable solution. The main contributions of this work are:

1. AFCS transmission and reception algorithm and mathematical details, considering wireless fading channel model and receive diversity.
2. Analysis of the effect of diversity and Maximum Ratio Combining (MRC) technique on the MSE performance for the number of receive antennas $N = 1, 2, 4, 6$.
3. Comparison of the wireless AFCS system's spectral efficiency for the different number of antennas ($N = 2, 4, 6$) with DCS with Phase Shift Keying (PSK) modulation scheme.

Following is an outline of the paper's content. The AFCS system architecture is explained in section 2. Section 3 discusses the detailed mathematical background, AWGN channel model, optimization algorithm, and mathematical analysis including fading channel model and diversity combining technique. In section 4, performance characteristics of AFCS are analyzed. Results and discussion is presented in section 5. Finally, section 6 provides the conclusion.

2. SYSTEM ARCHITECTURE

The AFCS schematic is shown in Fig. 1. It shows a system for the transmission of discrete time continuous amplitude memory-less Gaussian source over a wireless channel. The system is assumed to have a forward channel (ChF) and a feedback channel TxT2-ChR-RxR2. These channels connect the analog transmitting unit to the base station (receiver). We assume that the forward channel is a flat Rayleigh fading channel and the feedback channel (ChR) is a high-quality AWGN channel. A high quality noiseless feedback channel can be ensured by increasing the power transmitted from the receiver on the feedback channel. We presume the absence of memory for both channels. The AWG noise ζ_t in the forward channel is assumed to have a variance σ^2_{ζ}. The variance of feedback errors at the subtraction unit in the transmitting unit is considered to be σ^2_{v}. Without sacrificing generality, we assume that the source generates samples with a zero mean and variance σ^2_{0}.

In the overall system, the transmitting unit consists of a source, sample and hold circuit (Sampler), sub- traction unit, modulator (M1), and radio frequency (RF) transmitter module in the forward path. Along with this, it has a radio front-end receiving module and a demodulator (included in front-end receiver) as shown in Fig. 1 as a part of the Base Station (BS) unit. Additionally, BS unit has an estimator module (EM) in the forward path and a modulator and radio frequency transmitting system (TxT2) in the feedback path.

During the time interval T, each sample of the input signal is maintained at the subtraction unit's input and transmitted in n cycles (iterations), independent of the previous samples. BS unit analyses the signal received from the transmitting unit in each k^{th} cycle ($k = 1, 2, \dots, n$) and computes the sam ple's intermediate estimate in the estimator module (EM) and saves this estimate till the next cycle. The control signal that is conveyed to the transmit- ting unit via the feedback channel is also computed by EM. The maximum number of cycles or iterations in which one sample of input can be sent is $n = T/(\Delta t_0) = F_0/F$ where $T = 1/2F$ is the sampling period and $\Delta t_0 = 1/(2F_0)$ is the duration of one cycle of transmission. The minimum bandwidth re- quired of the forward channel is $F_0 = 1/(2\Delta t_0)$ and the bandwidth of the feedback channel is assumed to be greater than F_0.

For each k^{th} cycle of transmission $(k = 1; 2; ...; n)$;

$$e_k = x_k - \hat{B}_k = x_k - \hat{x}_{k-1} + v_k, \qquad (1)$$

where v_k is feedback error (AWGN) with variance σ^2_v. The transmitting unit modulator uses a double side band suppressed carrier pulse amplitude modulator (PAM). Every cycle, the values provided by the solution of the optimization problem are used to set the adaptive modulation depth of the modulator M_k. The transmitter has a nonlinear transfer function. The saturation appears if the signal $M_k e_k$ surpasses the carrier amplitude's saturation level. The transmitted signal is given by:

$$y_k = A_0 \begin{cases} M_k e_k, & \text{if } M_k|e_k| \le 1, \\ sign(e_k) & \text{if } M_k|e_k| > 1, \end{cases} \qquad (2)$$

and the received signal is given by:

$$\tilde{y}^r_k = h y_k + \zeta_k, \qquad (3)$$

where h is the fading coefficient and k is the Gaussian noise in the forward channel. $\tilde{y}^r_k$ is demodulated and the demodulated signal is given by:

$$\tilde{y}_k = A_0 h \begin{cases} M_k e_k + \zeta_k, & \text{if } M_k|e_k| \le 1, \\ sign(e_k) + \zeta_k, & \text{if } M_k|e_k| > 1. \end{cases} \qquad (4)$$

The signal after demodulation is routed to the estimator unit of BS. The estimator unit calculates the current estimate $\hat{x}_k$ using the Kalman type equation (5) which determines whether the contribution of the previous estimate or the contribution of the current observation will be greater in calculating the current estimate based on the value of L_k, set by the estimator unit.

$$\hat{x}_k = \hat{x}_{k-1} + L_k \tilde{y}_k, \qquad (5)$$

L_k controls how quickly estimates $\hat{x}_k$ converge to the transmitted sample's original value x_t. The value of L_k is calculated by the optimization task. The transmission unit receives the most recent estimate via the feedback channel. The transmission unit compares the received signal on the feedback channel to the original sample value and, the difference is sent to the modulator. The EM unit sets the value of L_k to L_{k+1} and modulation index M_k to M_{k+1} and the next $(k + 1)^{th}$ cycle begins. After n cycles, the final estimate $\hat{x}_n$ of the sample x_t is routed to the addressee and the next sample transmission begins. The initial values $\hat{B}_1$ and $\hat{M}_1$ are established by the saturation factor α which determines the allowable level of the likelihood of transmitter saturation, the variance of the input signal σ^2_0, and the mean value x0 of the input signal respectively.

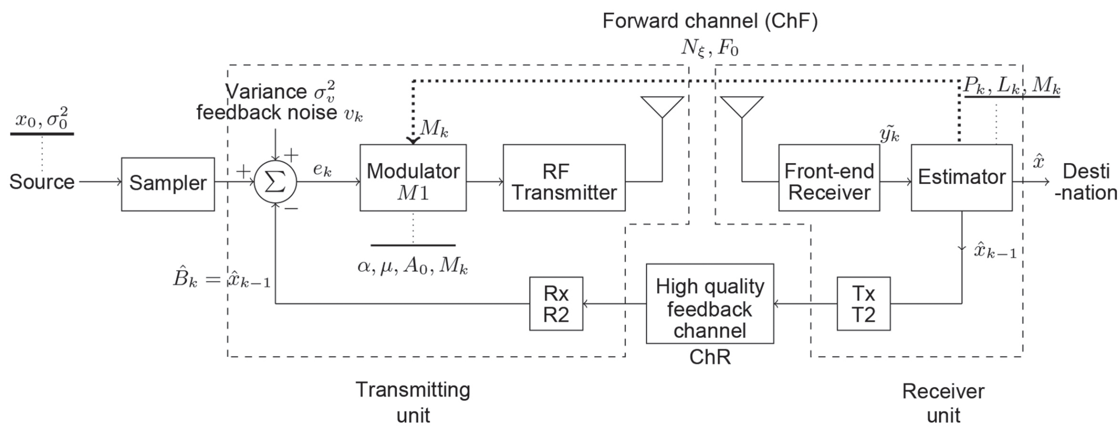


Fig. 1. System architecture of wireless Analog Feedback Communication

3. AFCS OPTIMIZATION FOR RAYLEIGH FADING CHANNEL

The MSE of current sample estimates can be calculated at the base station as the prior distribution of the input is assumed to be known at the receiver and the algorithm mentioned in section 2 provides the values of the current estimates. MSE in the k^{th} iteration denoted by P^k is given by:

$$P_k = E[(x - \hat{x}_k)^2]. \qquad (6)$$

MSE depends not only on the estimation algorithm but also on the parameters B_k and M_k of the adaptive modulator at the transmitter.

Hence we need to set the values of L_k; B_k; M_k to achieve minimum MSE. This in turn results in the optimization of both the transmission and reception algorithm. However, due to mathematical complications created by the saturation form of the modulator characteristic (2), a direct solution to this optimization problem is unattainable. Although, if a new transmission quality parameter – the permitted probability of over-modulation i.e. the probability of the appearance of errors due to over-modulation is introduced, this problem can be solved.

Probability of over-modulation of the adaptive modulator denoted by Pr^{over}_k in each iteration is given by:

$$Pr_k^{over} = Pr(M_k e_k > 1|\tilde{y}_1^{k-1}, B_{k-1}, M_{k-1}) < \mu. \quad (7)$$

Parameters (B_k; M_k) satisfying the inequality given in (7) form a permissible set of parameters Ω_k which guarantee the fulfillment of the condition $M_k|e_k| \leq 1$ with probability not smaller than 1-μ, where μ is the probability of modulator saturation and is the measure of errors occurring due to loss of information when the modulator saturates. Hence, the quantity $1-(1-\mu)^n \approx n\mu$ represents the frequency of appearance of errors caused by over-modulation at the time of sample transmission and is equivalent to bit-error rate (BER) in digital communication systems. Usually, values of between $10^{-12} \leq \mu < 10^{-4}$ is practically enough for the design. Considering this value of μ and (7), the parameters of adaptive modulator B_k; M_k are calculated. Such an adaptive modulator is known to be a statistically fitted adaptive modulator.

The expression for M_k as proposed in [21] is a result of consideration of this statistical fitting condition and is given by:

$$M_k = \frac{1}{\alpha\sqrt{P_{k-1} + \sigma_v^2}}, \quad (8)$$

here α is the saturation factor which takes into account the probability of over-modulation and the statistical fitting of the modulator. The relation between α and μ is given by:

$$\Phi(\alpha) = \frac{1}{\sqrt{2\pi}} \int_0^\alpha e^{\frac{-x^2}{2}} dx \geq \frac{1-\mu}{2}, \quad (9)$$

where $\Phi(\alpha)$ is known as Gaussian error function.

The statistically fitted modulator almost invariably operates as a linear unit, and the non-linear transmitter model (2) can be substituted by the linear one:

$$y_k = A_0 \hat{M}_k e_k = A_0 \hat{M}_k (x_k - B_k), \quad (10)$$

which in turn allows to replace the model in (4):

$$\tilde{y}_k = A_0 h \hat{M}_k (x_k - B_k) + \zeta_k. \quad (11)$$

The differences in the working of the model (2), (4) and the statistically fitted AFCS constructed using models, (10), (11) may emerge with a probability of $n\mu$. As a result, the MSE of the two systems might only vary by $O(n)$ order. However, the modulator's statistical fitting enables the transformation of a nonlinear optimization problem into a linear one. This task can then be solved using the Bayesian estimation theory approach [21],[22].

3.1. OPTIMIZATION OF RAYLEIGH FADING CHANNEL BASED ON MSE PERFORMANCE

As the fading coefficient is a complex quantity, the received signal and estimate signal given by (5) will also be complex and the MSE P_k takes the form:

$$P_k = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^*]. \quad (12)$$

Substituting (5) into equation (12), we have

$$P_k = E[(x_k - \hat{x}_{k-1} - L_k\tilde{y}_k)(x_k - \hat{x}_{k-1} - L_k\tilde{y}_k)^*]. \quad (13)$$

By differentiating P_k w.r.t. L_k^* and equating it to zero, we find the optimal value of L_k which minimizes the MSE:

$$L_k = \frac{E[(x_k - \hat{x}_{k-1})\tilde{y}_k^*]}{E[\tilde{y}_k\tilde{y}_k^*]}. \quad (14)$$

On substitution from (11), simplification and considering $E[(x_k - \hat{x}_{k-1})] = 0$, we get:

$$E[(x_k - \hat{x}_{k-1})\tilde{y}_k^*] = A_0 M_k h^* P_{k-1}, \quad (15)$$

where $P_{k-1} = E[(x_k - \hat{x}_{k-1})(x_k - \hat{x}_{k-1})^*]$. Now as $hh^* = |h|^2$, $E\{v_k v_k^*\} = \sigma_v^2$, $E\{\zeta_k\zeta_k^*\} = \sigma_\zeta^2$

$$E[\tilde{y}_k\tilde{y}_k^*] = |h|^2 {A_0}^2 {M_k}^2 (P_{k-1} + \sigma_v^2) + \sigma_\zeta^2. \quad (16)$$

Hence from (14),(15) and (16) L_k is given by

$$L_k = \frac{A_0 M_k h^* P_{k-1}}{\|h\|^2 {A_0}^2 {M_k}^2 (P_{k-1} + \sigma_v^2) + \sigma_\zeta^2}. \quad (17)$$

Substituting (17) into (13), with (11) and considering MSE in the previous cycle as given by:

$$P_{k-1} = E[(x_k - \hat{x}_{k-1})(x_k - \hat{x}_{k-1})^*]. \quad (18)$$

After simplification, we get:

$$P_k = \frac{[\|h\|^2 {A_0}^2 {M_k}^2 \sigma_v^2 + \sigma_\zeta^2] P_{k-1}}{\|h\|^2 {A_0}^2 {M_k}^2 (P_{k-1} + \sigma_v^2) + \sigma_\zeta^2}. \quad (19)$$

It is evident from the equation (19) that the MSE of transmission decreases with increasing modulation depth M_k and is independent of the control B_k values.

As MSE is independent of control values B_k, the optimization task is simplified to searching for the maximum value M_k from the set of values that satisfy the condition (7).

From equation (8), it seems that M_k values do not depend on the observations hence, they can be independent of the channel model used. Another control i.e. B_k is the estimate $\hat{x}_{k-1}$ during the previous cycle:

$$B_k = \hat{x}_{k-1}(\tilde{y}_1^{k-1}) = \hat{x}_{k-1}. \quad (20)$$

To summarize, the algorithm adopted by AFCS which ensures optimum transmission reception and achieves minimum mean square error is given by:

1. Calculate the estimate at the receiver using equations: (5) and (17)
2. Altering the transmitting unit's adaptive modulator in accordance with equations (20) and (8).

Initial values considered for the iterative algorithm: $\hat{x}_0 = x_0$; $P_0 = \sigma_0^2$.

3.2. DIVERSITY COMBINING WITH AFCS

Now consider that the output of the transmitter is transmitted using a single antenna but the receiver has multiple antennas. Fig. 2. shows the general case for N receive antennas. Let us consider $N = 2$.

The transmitted signal takes two independent channels $h1$ and $h2$. At the receiver we have:

$$\tilde{y}_{ki} = A_0 h_i \hat{M}_k (x_k - B_k) + \zeta_{k_i}, i \in \{1, 2\}. \quad (21)$$

The received signals $\tilde{y}_{k1}$ and $\tilde{y}_{k2}$ can be combined at the receiver using one of the diversity combining techniques like: selection combining (SC), equal gain combining (EGC) or maximum ratio combining (MRC) [1]. In SC, simply the strongest link is selected for transmission. As a result, if $h1 > h2$, the combined sequence at the receiver is $\tilde{y}_{k(SC)} = \tilde{y}_{k1}$, otherwise $\tilde{y}_{k(SC)} = \tilde{y}_{k2}$ and the corresponding channel coefficient is $hSC = max(h_1; h_2)$, whereas the noise variance remains the same as σ^2_ζ.

EGC gives equal weightage to both the received sequences. Thus, $\tilde{y}_{k(EGC)} = \tilde{y}_{k1} + \tilde{y}_{k2}$ and $h_{EGC} = h_1 + h_2$ give the received sequence and equivalent channel coefficient respectively. Due to equally adding both received signals, the noise variance becomes $2\sigma^2_\zeta$.

Finally, MRC blends the two sequences weighted by a fading coefficient factor. MRC is known to outperform SC and EGC and is known to be an optimum combining technique. The SNR improvement offered by MRC is better compared to SD and EGC [23].

3.3. MRC DIVERSITY COMBINING WITH AFCS

We find the effect of having receiver diversity (two receiving antennas) and MRC combining at the receiver on the equations and algorithm mentioned in section (3).

The received signal after MRC combining is given by:

$$\tilde{y}_{k(MRC)} = w_1{}^* \tilde{y}_{k1} + w_2{}^* \tilde{y}_{k2}, \quad (22)$$

where $w_1 = \frac{h_1}{\sqrt{|h_1|^2 + |h_2|^2}}$ and $w_2 = \frac{h_2}{\sqrt{|h_1|^2 + |h_2|^2}}$ are weights assigned to each channel (h_1 and h_2). The signal received on channel with fading coefficient h_1 is $\tilde{y}_{k1} = h_1 A_0 M_k (x_k - \hat{x}_{k-1} + v_k) + \zeta_{1k}$ and signal received on channel with fading coefficient h_2 is $\tilde{y}_{k2} = h_2 A_0 M_k (x_k - \hat{x}_{k-1} + v_k) + \zeta_{2k}$. Here ζ_{1k} and ζ_{2k} are uncorrelated AWG noises in the two diversity channels. Now this received and combined signal will be given to the estimator which calculates the estimate same as in section (2) equation (5).

$$\hat{x}_k = \hat{x}_{k-1} + L_k \tilde{y}_{k(MRC)}. \quad (23)$$

Similar to how the expressions of L_k and P_k were evaluated in section (3), new expressions for L_k and P_k will be evaluated with the received and combined signal $\tilde{y}_k(MRC)$. Hence substituting $\tilde{y}_k(MRC)$ in equation (14) in place of $\tilde{y}_k$, we get:

$$L_k = \frac{E[(x_k - \hat{x}_{k-1})\tilde{y}^*_{k(MRC)}]}{E[\tilde{y}_{k(MRC)}\tilde{y}^*_{k(MRC)}]}. \quad (24)$$

From equation (22) and following the same steps as in section 3.1 we get:

$$L_k = \frac{A_0 M_k P_{k-1}(w_1 h_1^* + w_2 h_2^*)}{E[\tilde{y}_{k(MRC)}\tilde{y}^*_{k(MRC)}]}, \quad (25)$$

where denominator term is given by:

$$\begin{aligned} &E[\tilde{y}_{k(MRC)}\tilde{y}^*_{k(MRC)}] \\ &= \sigma_\zeta^2(|w_1|^2 + |w_2|^2) \\ &+ A_0{}^2 M_k{}^2 (P_{k-1} + \sigma_v^2)(|w_1|^2|h_1|^2 + |w_2|^2|h_2|^2 \\ &+ w_1^* h_1 w_2 h_2^* + w_1^* h_1 w_2^* h_2), \end{aligned} \quad (26)$$

and,

$$\begin{aligned} &P_k = \\ &P_{k-1} - L_k{}^* A_0 M_k P_{k-1}(w_1 h_1^* + w_2 h_2^*) \\ &- L_k A_0 M_k P_{k-1}(w_1^* h_1 + w_2^* h_2) \\ &+ |L_k|^2[\sigma_\zeta^2(|w_1|^2 + |w_2|^2) \\ &+ A_0{}^2 M_k{}^2(P_{k-1} + \sigma_v^2)(|w_1|^2|h_1|^2 + |w_2|^2|h_2|^2 \\ &+ w_1{}^* h_1 w_2 h_2^* + w_1 h_1^* w_2^* h_2)]. \end{aligned} \quad (27)$$

4. PERFORMANCE EVALUATION

The performance of a communication channel in general depends on two factors: the properties of the channel and the properties of the source. When the smallest number of bits required to represent the source equals the highest number of bits possible on the channel, we reach the theoretically achievable the best performance limit (OPTA). OTPA can be calculated by finding the distortion rate function *DRF = D(R = C)* at a rate equal to channel capacity. It may appear simple to evaluate the DRF at a rate equal to the channel capacity; however, the issue is that the DRF rate is defined in bits per source sample, whereas the channel capacity is specified in bits per channel use. Since in AFCS, we are transmitting one source sample in *n* number of cycles i.e. *n*, channel uses, the capacity must be scaled accordingly. According to rate distortion theory, [24, 25], the output signal to distortion ratio (SDR) for a Gaussian source is given by:

$$\begin{aligned} D &= 2^{-2R}\sigma_x^2, \\ SDR &= \left(\frac{\sigma_x^2}{D}\right) = 2^{2R}. \end{aligned} \quad (28)$$

The OTPA equals:

$$OTPA = D(R)|_{R=KC/N} = D\left(\frac{KC}{N}\right). \quad (29)$$

In the case of AFCS, we are sending one sample in n cycles hence $K = n$ and $N = 1$, so the OTPA becomes:

$$OTPA = D(R)|_{R=nC} = D\left(nC\right). \quad (30)$$

The capacity of AFCS with a slow fading channel is found next.

4.2. CHANNEL CAPACITY

For a flat fading channel model with perfect channel knowledge at the receiver, the capacity of forward single input single output (SISO) channel is given by [1, 26]:

$$\begin{aligned} C &= \log_2(1 + SNR) \\ &= \log_2\left(1 + \frac{A_0^2 M_k^2 |h|^2 (P_{k-1} + \sigma_v^2)}{\sigma_\zeta^2}\right), \end{aligned} \quad (31)$$

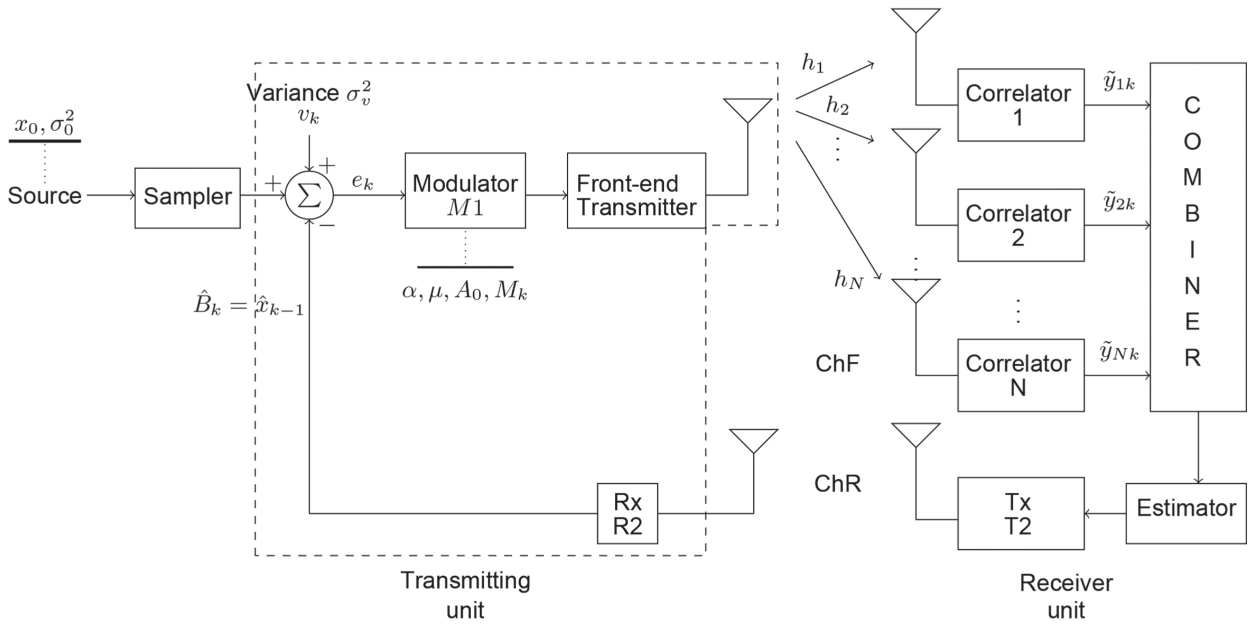


Fig. 2. System architecture of wireless AFCS with receive diversity

Here capacity is expressed in *bits/s/Hz*. Here h is considered to be a flat fading complex channel impulse response of block fading type. Even though h is constant for a block of transmitted symbols but as it is random, the channel capacity also becomes random. Hence ergodic capacity and outage probability make more sense in the case of fading channels. The ergodic capacity is defined as the statistical average of the mutual information where the expectation is taken over $|h|^2$ [26, 23] and is given by:

$$C_{erg} = E\left\{ \log_2 \left(1 + \frac{A_0^2 M_k^2 |h|^2 (P_{k-1} + \sigma_v^2)}{\sigma_\zeta^2}\right) \right\}. \quad (32)$$

By Jensen's inequality [1] applied to (32):

$$\begin{aligned} &E\left\{ \log_2 \left(1 + \frac{A_0^2 M_k^2 |h|^2 (P_{k-1} + \sigma_v^2)}{\sigma_\zeta^2}\right) \right\} \\ &\leq \log_2 \left(1 + \frac{A_0^2 M_k^2 E\{|h|^2\} (P_{k-1} + \sigma_v^2)}{\sigma_\zeta^2}\right), \end{aligned} \quad (33)$$

where E{.} stands for expectation. Finally, the OTPA considering optimum AWGN capacity and DRF given by (28) is:

$$\begin{aligned} SDR &= \left(\frac{\sigma_x^2}{D}\right) = 2^{2nC} \\ SDR_{dB} &= \left(\frac{\sigma_x^2}{D}\right)_{dB} = 20n \log_{10}\left(1 + Q_F^2\right) \end{aligned} \quad (34)$$

5. RESULTS AND DISCUSSION

Based on the results found in section 3 and section 4, the algorithm pertaining to the Rayleigh fading channel model is implemented and simulation is performed in MATLAB. AFCS in Rayleigh fading channel with single input single output (SISO) without diversity is compared with the system with receive diversity with N number of receive antenna ($N = 2; 4; 6$) is investigated and the results are presented.

For simulation, a random input signal with mean x_0 = 1, variance σ^2_0 = 0.625, and band-limited to 2.5*kHz* is taken into account which is sampled at a frequency of 8*kHz*. The plots are shown in Fig. 3 to Fig. 6.

MSE between the input and estimate is considered to be the overall performance criteria. Fig. 3 shows MSE per iteration for SISO Rayleigh fading channel, and SIMO Rayleigh fading channel with MRC for 2,4, and 6 receiving antennas. Convergence and MSE performance comparison for two different channel signal-to-noise Ratio (SNR) values 5*dB* and 10*dB* can be observed in Fig. 3a and 3b respectively. MSE performance is better for 10dB for obvious reason of higher channel SNR. The evident improvement with the inclusion of receive diversity can be seen in both cases. For all the schemes the MSE for the proposed communication system architecture (Analog Feedback Communication System) converges to zero. The values of MSE in SISO case during the first iteration is higher compared to SIMO case, and it decreases as the number of diversity paths increases. Furthermore, With an increase in the number of receive antennas, the MSE per iteration decreases, and the convergence to zero MSE happens in fewer iterations compared to SISO Rayleigh fading channel scheme. Fig. 4 shows the plot showing the variation in AFCS output SNR for different channel SNR. It shows an evident improvement in SNR at channel output with receive diversity. The improvement in SNR is about 15dB at 20 dB channel SNR in the case of 6 receive antennas compared to the SISO channel. This improvement in SNR certainly leads to the improvement in approaching the capacity limit. Fig. 5 shows the maximum achievable capacity limits for AWGN AFCS compared to Rayleigh fading AFCS. The graph is plotted using (32). The maximum achievable performance limit of Rayleigh fading AFCS is less than that of AWGN. The gap can be made small by using spatial diversity and MIMO techniques. Spatial diversity in terms of multiple receive antennae to mitigate the effect of fading channels is the subject of the current paper.

Finally, in Fig. 6a, the behavior of AFCS with Rayleigh fading channel and receive diversity is shown on the classic spectral efficiency versus bit energy to noise ratio plot for ideal systems. Here the operating points of M-ary PSK digital communication system are shown and are compared with the operating points of diversity-enabled AFCS with N=1,2,4,6 receive antennas. Diversity-enabled AFCS seems to coincide with ideal systems which achieve Shannon's capacity. The zoomed version of the plot in Fig. 6b shows that the spectral efficiency improves with an increase in the number of diversity branches.

5.2. COMPARISON AND DISCUSSION

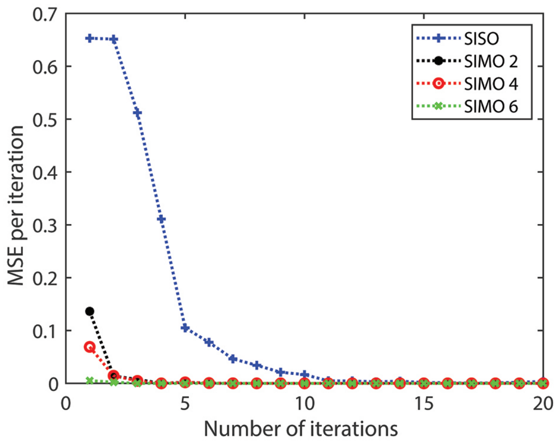
We proposed an analog communication system with the Rayleigh fading channel model in this paper. A one-to-one comparison with the current wide range of digital communication systems with advanced coding and decoding techniques does not make sense because the majority of the signal processing chain, and performance criteria focused is different in AFCS and DCS.

In AFCS MSE is the performance criteria and both transmitter and receiver are optimized to obtain minimum MSE as shown in section 3 Whereas in digital communication systems (DCS), there are multiple performance criteria that are correlated like bit-rate, bit-error-rate (BER), power-bandwidth efficiency, etc. The search for an optimum balance between these performance criteria involves tradeoffs. For example, increasing the bit rate decreases the BER, to achieve a good BER more power must be spent, and so on. Many stages are involved in DCS like digitization, source-channel coding/decoding, modulation, etc., and optimizing each of these processes for common performance criteria is practically very difficult and leads to a very complex, power-consuming system.

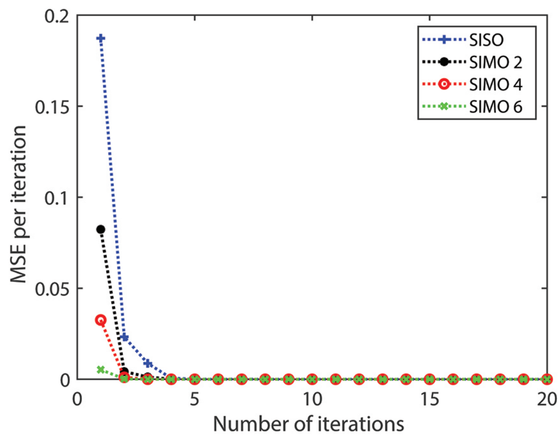
The AFCS uses feedback and iterative estimation algorithm instead of complex coding. Hence will be a better choice for applications that are power constrained such as wireless sensor networks, satellite communication, and many applications where battery-operated sensors transmit information to base stations. In applications where source information is analog, converting it to digital and losing the information in the process of quantization, incorporating complex coding and decoding algorithms to reach the performance limits seems a costly and complicated process. Instead, if we transmit uncoded analog information directly, and utilize the feedback which is available in almost all scenarios of communication these days, we can utilize the available communication resources efficiently. There is definitely an iterative process and feedback involved, but as shown in Fig. 3, with the diversity scheme, it takes hardly 2-3 iterations to reach close to zero mean square error (MSE). The mean square error quickly converges to zero as we increase the number of antennae (N).

In DCS a very low BER does not ensure that an analog signal will be transmitted with reliability. The quantization loss always will be present. Whereas in the case of AFCS transmission a very low or almost zero MMSE ensures reliable transmission.

The design of digital communication stages is done keeping in mind a particular channel SNR as a design parameter. If the channel conditions are poorer compared to that, then the performance of DCS drastically deteriorates but does not improve if the channel condition is better than the design parameter. This is known as the cliff effect, and DCS suffers from the cliff effect. In AFCS symbols are transmitted hence there is no problem of delay which occurs when long block lengths are used. The adaptive modulator is adjusted according to the channel conditions iteratively, hence there is no cliff effect problem and there is a graceful degradation in case of poor channel conditions.



(a) MSE per iteration for SNR 5dB



(b) MSE per iteration for SNR 10dB

Fig. 3. MSE per iteration for SISO Rayleigh fading channel, SIMO Rayleigh fading channel with MRC for 2 ,4 and 6 receiving antennas.

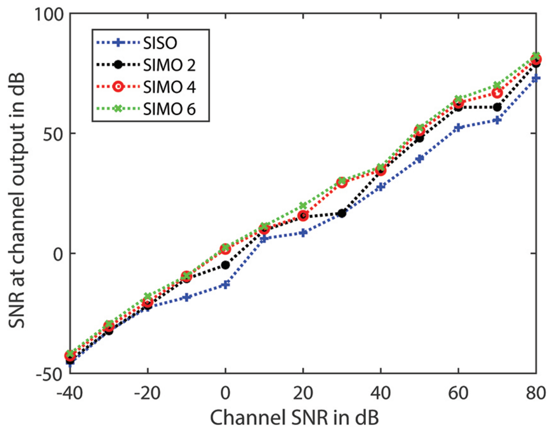


Fig. 4. Graph showing the variation in AFCS output SNR for different channel SNR

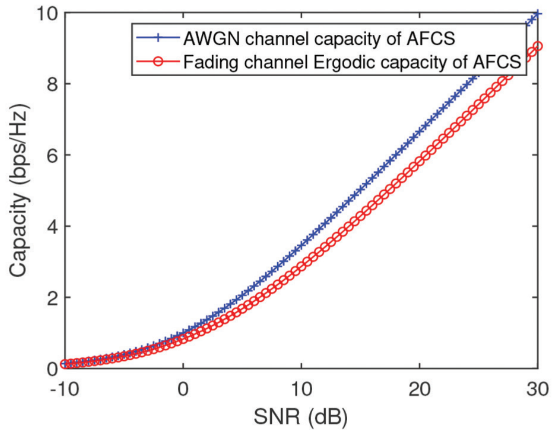
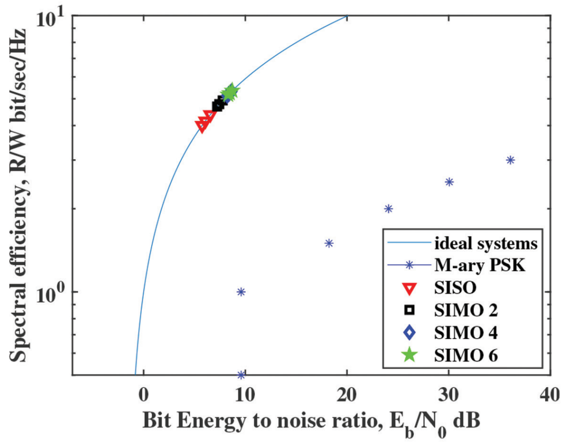
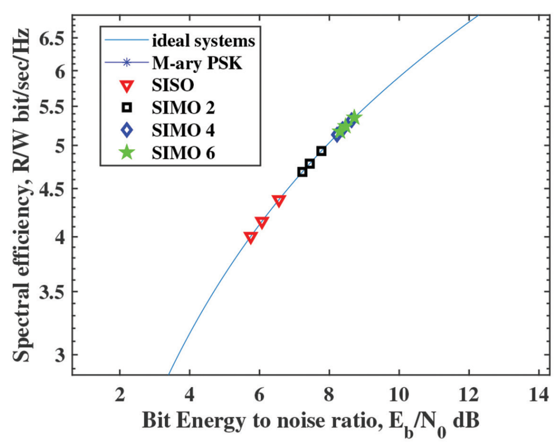


Fig. 5. Graph showing the comparison of maximum achievable capacity, in case of an AWGN AFCS channel and AFCS with fading channel model



(a) Spectral efficiency versus bit energy to noise ratio plot (normal scale)



(b) Zoomed in version

Fig. 6. Spectral efficiency versus bit energy to noise ratio

6. CONCLUSION

A system model is created and a thorough analysis is provided for AFCS with Rayleigh fading channel model and MRC receive diversity. It can be seen that the system can reach the Shannon capacity boundary which is something that most spectrally efficient digital communication systems cannot achieve. Because the AFCS system does not involve digitizing and coding, many of the procedures such as analog-to-digital conversion, encoding, digital-to-analog conversion, decoding, and so on are not required for AFCS. This leads to significant savings of energy which in turn results in the transmitter becoming lighter and more power efficient. Receive diversity used with MRC in AFCS results in enhanced MSE performance and improved power bandwidth efficiency. OTPA performance is also seen to improve with diversity. Since power requirements for short- to medium-range applications involving sensor networks are relatively strict, such a system could be effectively used in those applications. By using diversity strategies for AFCS, SNR and coverage area can both be further increased.

We are investigating more diversity strategies including MIMO channels in AFCS as future work. MIMO AFCS is anticipated to produce better outcomes by mitigating the effects of fading channels.

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