Illustration of the voltage stability by using the slope of the tangent vector component

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Agron Bislimi
UBT, University for Business and Technology,
St. Calabria n.n. 10000, Prishtina, Kosovo.
agron.bislimi@ubt-uni.net

Abstract *– This Paper is dedicated to the analysis of the evolution of the tangent vector during the Continuous Power Flow (CPF) iterations. The flow of the tangent slope (measured in degrees) is shown through the coefficient of lambda tangent vector component and the maximum voltage tangent vector component. A 17 Node Network was used for the purposes of this Paper. The system was modelled in MATLAB software. The admittance matrix of the node voltage equations was formulated and the functions in MATLAB were developed for the systematic formation of the node admittance matrix. Equations for the calculated network were generated in MATLAB. 32 Iterations were performed. Iterations and corrections of iterations were done manually. Firstly, the results for the tangent vectors calculated through the CPF program were compared to the results for the tangents directly calculated with mathematical formula for the tangent, and both results match. The chart, which contains the classical PV curve and the flow of tangent vectors during the CPF iterations, was developed based on the results obtained. The increase in the slope of the tangent in the PV diagram imposes a clear numerical stability limit by specifying an angle limit value, which can be used to trigger an alarm. In addition to the classic Power-Voltage (PV) curve, this serves as an additional indicator for ensuring voltage stability of the examined system.*

Keywords: *Voltage Stability, Tangent Slope, CPF Iterations, Tangent Vector Component*

1. INTRODUCTION

This Research differs from others because the literature uses the tangents to find the PV curves for the examined system, which serves as an indicator to ensure the voltage stability of the examined system. In addition to classical PV curves, my Research also explains the voltage stability through the slope of the tangent vector components during CPF iterations. This serves as an additional indicator for ensuring the stability of the voltage of the examined system. Using examples, it also provides a theoretical and practical explanation of the matching of directly calculated tangent results and their calculation through CPF.

The tangent vector component "t" in [1] represents the values (voltage angle δ, voltage magnitude V and load parameter lambda λ). The variable values with the largest tangent vector component from the tangent "t" are used as continuous parameter. The lambda tangent vector component ($d\lambda$) is positive for the upper part of the PV. It is zero at the critical point where the Jacobian matrix becomes singular and is negative at the bottom of the PV curve. The sign of $d\lambda$ shows us whether or not the critical point has been reached. The flow of the voltage magnitudes in view of the network load provides information about the voltage stability limit or the network load which, when exceeds, causes collapse to the voltage [1]. The most common parameterization techniques used by CPF to remove the singularity of the PF Jacobian matrix is in [2]. The sensitivity of the information discussed from the tangent vector. The continuation power flow traces the complete P-V curves by automatically changing the value of a parameter. In the local parameterization technique, a parameter change always occurs close to the Maximum Loading Point (MLP). Generally, the loading factor λ is an initially chosen parameter. Close to the MLP, it changes to the voltage magnitude that presents the largest variations and after a few points, it changes back to λ. The differential change in the voltage at each node for a given differential change in the system load is available from the tangent vector [2]. In [3] is explained the voltage stability index from the tangent vector. The ratio $Cd\lambda/dVi$, where C is constant, which is easier to be handled numerically, can be defined as voltage stability index for the entire system [3]. In each CPF iteration step, the active and reactive loads of the network are increased by a certain percentage of the base load. Voltage stability index using the tangent can be derived at [4]. The weakest node means a node which is nearer to the voltage collapse due to the lack of the reactive power. It means that the weakest node has the maximum ratio of differential change in the voltage to the differential change in the active power demand at the critical point. It represents the gradient or the tangent of a PV curve [5]. In

[6-8], it consists of the calculation of the power flow and voltages of a transmission network for specified terminal conditions. Such calculations are required for steady-state performance analysis of power systems. Thus, the tangent vector provides information about critical nodes or critical parts of the network [6-8]. Equation $[\underline{F\delta}, \underline{FV}, \underline{F\lambda}]$ is the Jacobian matrix of the power flow, augmented by a column multiplied by a tangent vector required. One problem arises while solving this Equation, i.e., the additional unknowns are added to the power flow equation when the load factor λ is added to the power flow equations. The tangent vector is represented by the Equation $[\underline{d\delta}, \underline{dV}, \underline{d\lambda}]^T$, T= transpose. The critical point is reached when the load change reaches its maximum value. The change of sign of dLambda in the tangent vector is constantly checked. The CPF method, which includes the process of predictor and the process of corrector, can be applied to solve the non-linear equations. The predictor process starts with the calculation of the tangent vector. The tangent vector must be normalized in order to guarantee the non-singularity of the augmented Jacobian matrix [9-11]. The tangent vector gives the sensitivity of the parameters at a point in the PV curve where they are evaluated. The sensitivities in the entire parts of the PV curve should be evaluated [12]. The tangent vector t is used to predict initial values for the next step of CPF. The tangent size is ($6n + 1$) for a three-phase system, where there are $6n$ voltage variables and one loading parameter (λ) variable. Prediction can be made by using tangent and secant predictors. Tangent predictors predict only by using the current solution, and the secant predictors use the current and previous solutions in order to make predictions [13]. An efficient geometric parameterization technique for the CPF is presented in [14]. Intending to reduce the CPU time, the effectiveness caused by updating the Jacobian matrix is investigated only when the system undergoes a significant change. The Paper presents in [15] a potential algorithm for Continuation Power Flow method in voltage stability assessment. [16] proposes an innovative method by modifying the Conventional Continuation Power Flow (CCPF) method. The tangent predictor is proposed to estimate the next predicted solution from two previous corrected solutions. And the corrector step is proposed to determine the next corrected solution on the exact solution. This correct solution is constrained to lie in the hyperplane running through the predicted solution orthogonal to the line from the two previous corrected solutions [16].

This field of research is currently focused on researching the voltage stability by using the slope of the tangent vector component. This Paper is structured as follows: Second chapter shows the methods employed, namely the direct calculation of tangents and tangent calculation through CPF. It consists of a research and detailed explanation of the Lambda as a continuous parameter. Moreover, it illustrates the Voltage V as a continuous parameter. Third section consists of the results. Furthermore, it compares the tangent vectors when calculated with the CPF program and when calculated directly using the mathematical formula. Finally, the chart, which contains the classical PV curve and the flow of the tangent vectors during the CPF iterations, is developed using the results. In addition to the classic Power-Voltage (PV) curve, this serves as an additional indicator for ensuring voltage stability of the examined system.

2. METHODS - DIRECT CALCULATION OF TANGENTS AND TANGENT CALCULATION THROUGH CPF. TANGENTS USING LAMBDA (λ) AND VOLTAGE V AS CONTINUOUS PARAMETERS

2.1. DIRECT CALCULATION OF TANGENTS AND TANGENT CALCULATION THROUGH CPF

2.1.1. Direct calculation of tangents

The method below was employed for the direct calculation of tangents using the tangent formula

To calculate the slope of any point on the line, we draw a tangent to it and calculate the value of tan of the angle it makes with the base. A tangent to a curve is a straight line that touches the curve at a given point and represents the gradient of the curve at that point.

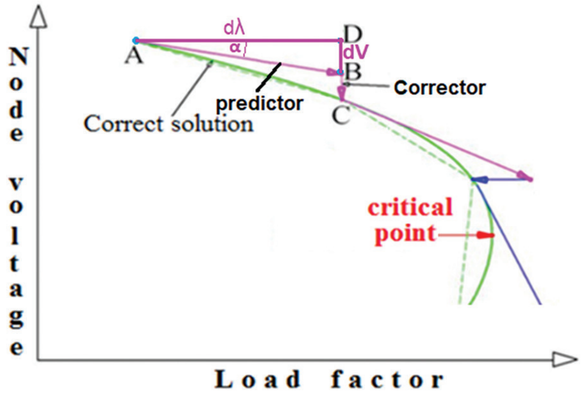

Fig. 1. Direct calculation the tangent in the 17-node network.

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} = \frac{DB}{AD} = \frac{dV}{d\lambda}$$

2.1.2. Calculation of the Tangent using CPF Predictor step

The method below was employed to calculate the tangents through CPF. The system of non-linear equations looks like this after inserting the lambda parameter:

$$F(\underline{v}, \underline{\delta}, \underline{\lambda}) = 0 \tag{1}$$

The Taylor linearization of equation (1) yields, neglecting the higher-order terms:

$$\underline{dF} = \underline{F_v} d\underline{v} + F_{\underline{\delta}} d\underline{\delta} + \underline{F_\lambda} d\lambda = 0$$

$F\underline{v}$- Derivatives of F with respect to the magnitude of the voltage. $F\underline{\delta}$- Derivatives of F with respect to the voltage angle. $F\underline{\lambda}$- Derivatives from F to λ, corresponds to a column vector [1].

The first task in the predictor step is to calculate the tangent vector component.

$$[\underline{F\delta},\ \underline{FV},\ \underline{F\lambda}]\begin{bmatrix}\underline{d\delta}\\ \underline{dV}\\ \underline{d\lambda}\end{bmatrix}=0 \tag{2}$$

On the left are Jacobian matrix elements

$$\underline{t}=\begin{bmatrix}\underline{d\delta}\\ \underline{dV}\\ \underline{d\lambda}\end{bmatrix} \tag{3}$$

$\underline{t}$ is the tangent vector component

The system of equations (2) is underdetermined by the additional unknown $d\lambda$.

One, therefore, adds another equation (5) and supplements the right-hand 0-vector by:

$$tk=\pm 1 \tag{4}$$

$$\begin{bmatrix}\underline{F\delta} & \underline{F\nu} & \underline{F\lambda}\\ \underline{e_k} & & \end{bmatrix}[\underline{t}]=\begin{bmatrix}\underline{0}\\ \pm 1\end{bmatrix} \tag{5}$$

$\underline{e}_k$ is a vector with all elements equal to 0, except the k^{th} element, which equals 1 [3].

The value of k depends on the continuation parameter. For instance, with the continuation parameter l, k corresponds to the last position in e_k. If the continuation parameter increases over the course of the CPF iterations, t_k is set to +1, otherwise, it is set to -1. For instance, with continuation parameter l, before the reversal point of the PV curve (the stability limit) is reached, $t_k = 1$, then $t_k = -1$.

Solving equation (5) gives the tangent vector component [t].

2.2. LAMBDA USED AS A CONTINUOUS PARAMETER AND VOLTAGE V AS CONTINUATION PARAMETER

2.2.1. lambda used as a continuous parameter

Since λ Continuation parameter is in e_k at the λ position (n-th position) is 1. As long as λ is continuation parameter, the e_k series and λ column is eliminated at the corrector step. (Since λ is fixed on the predictor value in the corrector iterations, it has to be eliminated from the corrector equation system). Chap 2. need to construct the classical PV curve. The iterations using the continuous parameter λ are performed while the tangent vector component λ is greater than the component of the high (negative) voltage tangent vector. As long as λ is a continuous parameter, λ remains constant during the corrector iterations, but the voltage angles δi and voltage values Vi change. Fig. 3 explains the tangents when the parameter λ is continuous $V=f(\lambda)$ for the 17-node network shown in Fig. 2 and explains in which range of angles the iterations span using the continuous parameters Lambda and V. For the angle range (0 → -45°), the parameter Lambda is a continuous parameter.

The largest voltage tangent vector component will be in range (0 → -1). The closer we get to the critical point, the higher the tangent angle. For the angle -44°, we gain -0.9656. For the angle -135°, the condition to change the continuous parameter V to the continuous parameter Lambda is met and the iterations are carried until the end by using the continuous parameter Lambda.

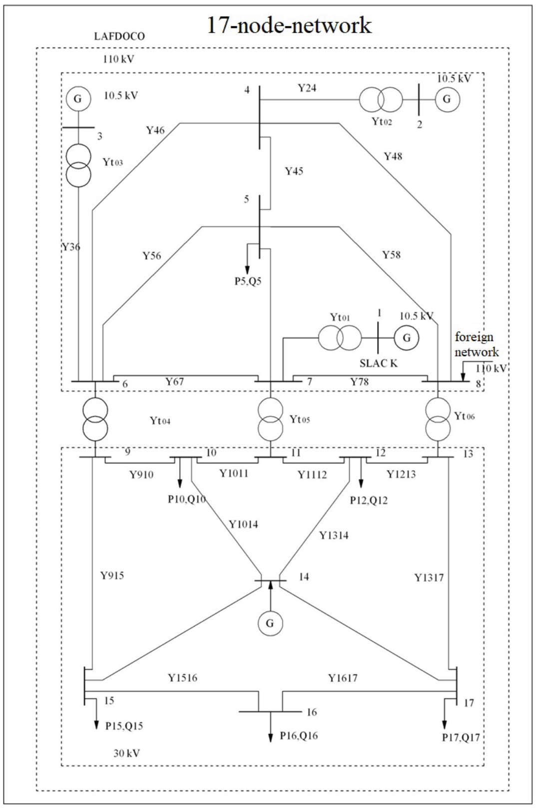

Fig. 2. 17- node network

tan (-135° → -180°) = (1→ 0). The largest (-1) voltage tangent vector component will be in the interval (-1→ 0), (the lower part of the curve).

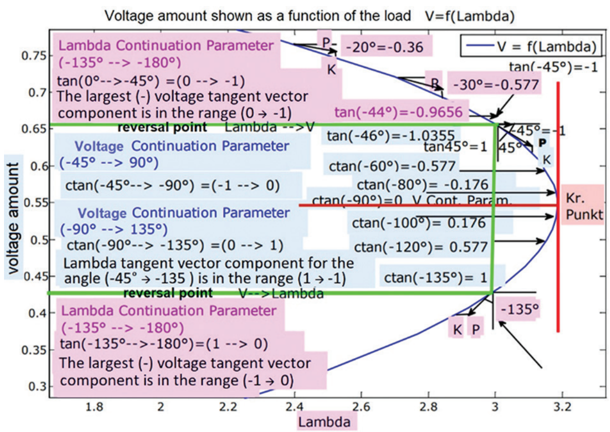

Fig. 3. Tangents using λ as a CP $V=f(\lambda)$

At the beginning of the CPF iteration, large changes in load correspond to small changes in voltage $d\lambda >> dV$. As the iterations progress, the ratio between changes in load and changes in voltage becomes smaller and smaller. These relations are expressed by the quotient between the largest negative voltage and the lambda tangent vector component.

Fig. 3 shows that the value of the largest negative voltage tangent vector component of -0.36 corresponds to an angle of $tan^{-1}(-0{,}36)$ = -20°. If the condition of inverting λ to V is met, then the largest voltage tangent vector component value is -1. This corresponds to a tangent angle of -45 degrees.

Thereafter, the absolute value of the quotient between the largest voltage tangent vector component and the λ tangent vector component is greater than 1 (negative sign), resulting in a tangent angle of more than -45 degrees.

2.2.2. Voltage V as continuation parameter

As long as V is continuation parameter, the e_k- vector row and the V_5- column are eliminated at the corrector step. (V_5 is fixed at the predictor value in the corrector iterations). The differentials $dV_5 = 0$ are thus eliminated from the corrector equation system. For the illustration in Fig. 4, the tangent angle is calculated from the quotient between λ and the largest voltage tangent vector component. Since the voltage component is fixed at -1, it follows directly from the p. u. value of the lambda tangent vector component. The change from continuation parameter λ to V occurs at a voltage tangent vector component of -1.0355 p. u., which corresponds to a tangent angle of -46°. Reason: The inversion condition is that tangent angle is < -45°. The closer you get to the critical point, the more the relationship between voltage and load changes. This corresponds to a tangent angle of -90° at the critical point in Fig. 4.

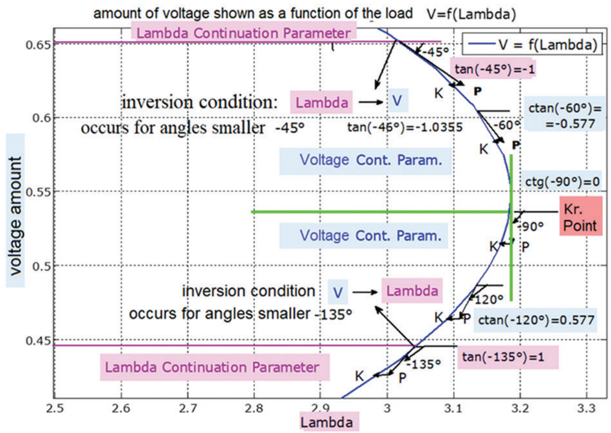

Fig. 4. $V=f(\lambda)$ using V as a continuous parameter

Thereafter, the sign of the lambda tangent vector component reverses from + to -, and the tangent angle assumes values of more than 90°. The sign remains negative, see Fig. 3, lower part of the PV curve. The values of both the lambda tangent vector component and the tangent angle increase again. Because the value of the tangent angle is negative, the slope of the tangent becomes flatter again.

For an angle of -135°, the quotient between voltage and lambda tangent vector components becomes 1 (tan(-135) = 1), and thus the condition to invert the continuation parameter from V back to λ is met. Once this condition is met, the changes in the voltage become smaller again.

Lambda tangent vector component for the angle range (-45° → -135°) will be in the range (1→ -1). For the angle -45° (tan(-45°) = -1), the condition of changing the continuous parameter Lambda to the continuous parameter V is met again.

After that, CPF iterations continue with V as continuous parameter.

- For the angle (-46°)→tan(-46°)=-1.0355.
- For the angle -80° → $ctan$(-80°) = -0.176 (The upper part of the curve)
- For the angle (-90°) → $ctan$(-90°)=0, ($tang$(-90°)=Infinite).
- For the angle (-100°) → $ctan$(-100°)=0.176 (The bottom part of the curve)

3. RESULTS: COMPARISON OF THE TANGENT VECTORS WHEN CALCULATED USING THE CPF PROGRAM AND WHEN CALCULATED DIRECTLY. COURSE OF THE QUOTIENT OF THE LARGEST (NEGATIVE) VOLTAGE TANGENT VECTOR COMPONENT AND LAMBDA TANGENT VECTOR COMPONENT.

3.1. COMPARISON OF THE TANGENT VECTORS WHEN CALCULATED USING THE CPF PROGRAM AND WHEN CALCULATED DIRECTLY

3.1.1. Direct calculation of tangents

Data for direct calculation the tangent for the first iteration.

Iteration 1: Before the predictor: V_5=0.9520
Before the predictor: λ=0
After the predictor: λ=0.15
$dV1 = V_{5\ (\text{After the predictor})} - V_{5\ (\text{Before the predictor})} =$
=0.9520-0.9442=-0,0078
$d\lambda 1 = \lambda_{(\text{After the predictor})} - \lambda_{(\text{Before the predictor})} = 0.15 - 0 = 0.15$

The direct calculation of tangents for the first iteration is:

$$t\,1 = \frac{\mathrm{dV1}}{\mathrm{d\lambda 1}} = \frac{-0.0078}{0.15} = -0.052$$

Data for direct calculation the tangent for iteration 16.

Iteration 16: Before the predictor: V5=0.5541
After the predictor: V5=0.5441;
Before the predictor: λ=3.1854;
After the predictor: λ=3.1880;
$dV1 = V_{5\ (\text{After the predictor})} - V_{5\ (\text{Before the predictor})}$ =0.5441 - 0.5541=-0,01
$d\lambda 1 = \lambda_{(\text{After the predictor})} - \lambda_{(\text{Before the predictor})}$ = 3.1880 – 3.1854= 0.0026

$$\text{ct }1 = \frac{1}{\text{t1}} = \frac{\text{d}\lambda 1}{\text{dV1}} = \frac{0.0026}{-0.01} = -0.26$$

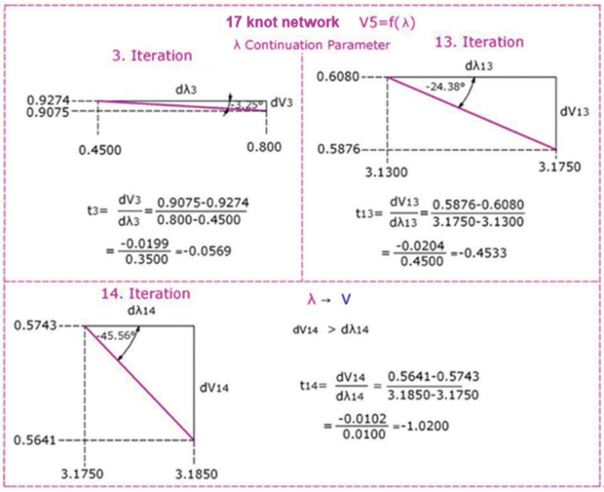

Fig. 5. Direct calculation of the tangents in the iterations 3, 13, 14 at the node 5 in the 17-node network.

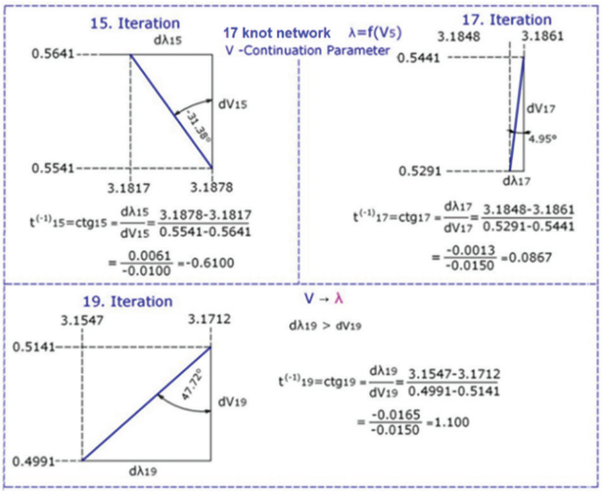

Fig. 6. Direct calculation of the tangents in the iterations 15, 17 and 19 at node 5 in the 17-node network.

3.1.2. Tangent calculation through CPF and compare

Calculation of tangents through CPF:

The tangent vector components for a 17-node network calculated using the CPF are as follows:

In the iteration 1:

$$t_1 = \begin{array}{cccccc} \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 & \delta_7 \\ [\,0.0031 & -0.0023 & -0.0318 & -0.0631 & -0.0370 & -0.0360 \\ \delta_8 & \delta_9 & \delta_{10} & \delta_{11} & \delta_{12} & \delta_{13} \\ -0.0303 & -0.0531 & -0.0573 & -0.0504 & -0.0562 & -0.0495 \\ \delta_{14} & \delta_{15} & \delta_{16} & \delta_{17} & & \\ -0.0578 & -0.0632 & -0.0724 & -0.0598 & & \\ V_4 & V_5 & V_6 & V_7 & V_8 & V_9 \\ -0.0303 & -0.0531 & -0.0573 & -0.0504 & -0.0562 & -0.0495 \\ V_{10} & V_{11} & V_{12} & V_{13} & V_{14} & V_{15} \\ -0.0303 & -0.0531 & -0.0573 & -0.0504 & -0.0562 & -0.0495 \\ V_{16} & V_{17} & \lambda & & & \\ -0.0562 & -0.0495 & \mathbf{1.0000}]^T & & & \end{array}$$

T = transpose

In the iteration 1, the largest (negative) voltage tangent vector component occurs at node 5. The tangent inclination in direction V_5 is given by the quotient $dV_5/d\lambda$ =-0,0521/1 = -0,0521. If you compare this with the result of the direct calculation, which is 0.052 (see subchapter 3.1.1), you can see a good agreement with an error of only 0.001.

In the iteration 16, the tangent direction for the continuation parameter V_5 results from the values taken from $t16$ to $d\lambda/dV_5$ = -0.2626/1= -0.2626.

If you compare this with the result of the direct calculation, which is 0.26, you can see a good agreement with an error of only 0.0026.

Table 1. Tangents in the iterations 1-32 at the node 5 in the 17-node network calculation through CPF.

Tangents in the iterations 1-32 at the node 5 in the 17-node network.			
Iteration	dV_5	$d\lambda$	$tangent{=}dV/d\lambda$
1	-0.0521	1.0000	-0.0521
2	-0.0536	1.0000	-0.0536
3	-0.0570	1.0000	-0.0570
4	-0.0616	1.0000	-0.0616
5	-0.0684	1.0000	-0.0684
6	-0.0776	1.0000	-0.0776
7	-0.0912	1.0000	-0.0912
8	-0.1103	1.0000	-0.1103
9	-0.1474	1.0000	-0.1474
10	-0.1949	1.0000	-0.1949
11	-0.2440	1.0000	-0.2440
12	-0.3115	1.0000	-0.3115
13	-0.4523	1.0000	-0.4523
14	-1.0238	1.0000	-1.0238
15	-1.0000	0.6133	-0.6133
16	-1.0000	0.2626	-0.2626
17	-1.0000	-0.0851	0.0851
18	-1.0000	-0.5960	0.5960
19	-1.0000	-1.0975	-1.0975
20	-0.6314	-1.0000	0.6314
21	-0.4746	-1.0000	0.4746
22	-0.3662	-1.0000	0.3662
23	-0.2818	-1.0000	0.2818
24	-0.2255	-1.0000	0.2255
25	-0.1827	-1.0000	0.1827
26	-0.1377	-1.0000	0.1377
27	-0.1157	-1.0000	0.1157
28	-0.1021	-1.0000	0.1021
29	-0.0927	-1.0000	0.0927
30	-0.0856	-1.0000	0.0856
31	-0.0802	-1.0000	0.0802
32	-0.0759	-1.0000	0.0759

Iteration 3, 13 and 14 using continuation parameter lambda

The tangent vector components for a 17-node network calculated using the CPF are as follows:

In the iteration 3:

$t_3=$	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7
	[0.0007	-0.0049	-0.0349	-0.0685	-0.0403	-0.0383
	δ_8	δ_9	δ_{10}	δ_{11}	δ_{12}	δ_{13}
	-0.0326	-0.0569	-0.0612	-0.0539	-0.0600	-0.0531
	δ_{14}	δ_{15}	δ_{16}	δ_{17}		
	-0.0618	-0.0673	-0.0770	-0.0639		
	V_4	V_5	V_6	V_7	V_8	V_9
	-0.0321	**-0.0570**	-0.0294	-0.0306	-0.0368	-0.0219
	V_{10}	V_{11}	V_{12}	V_{13}	V_{15}	V_{16}
	-0.0171	-0.0219	-0.0182	-0.0242	-0.0195	-0.0299
	V_{17}	λ				
	-0.0562	**1.0000**]^T				

T = transpose

Node 1 is the slack, nodes 2, 3 and 14 are PV nodes. Therefore, no voltage magnitude components appear for these nodes in the tangent vector.

The largest (negative) voltage tangent vector component occurs at node 5. The tangent inclination in direction V_5 is given by the quotient $dV_5/d\lambda$ = -0,0570/1 = -0,057. If you compare this with the result of the direct calculation from Fig. 5, which is 0.0569, you can see a good agreement with an error of only 0.0001.

In the iteration 13, the tangent vector components calculated using the CPF are:

$t_{13}=$	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7
	[-0.3010	-0.3102	-0.3580	-0.6216	-0.3646	-0.2516
	δ_8	δ_9	δ_{10}	δ_{11}	δ_{12}	δ_{13}
	-0.2733	-0.4006	-0.4024	-0.3677	-0.3962	-0.3767
	δ_{14}	δ_{15}	δ_{16}	δ_{17}		
	-0.4049	-0.4184	-0.4398	-0.4080		
	V_4	V_5	V_6	V_7	V_8	V_9
	-0.2624	**-0.4523**	-0.2394	-0.2761	-0.3071	-0.14131
	V_{10}	V_{11}	V_{12}	V_{13}	V_{15}	V_{16}
	-0.1083	-0.1684	-0.1189	-0.1725	-0.0887	-0.1120
	V_{17}	λ				
	-0.0958	**1.0000**]^T				

T = transpose

The comparison of dV_5 = -0.4523 with the value of -0.4533 from Fig. 5 shows again good value agreement.

The same applies to the iteration 14:

$t_{14}=$	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7
	[-0.7464	-0.7545	-0.8245	-1.4313	-0.8271	-0.5460
	δ_8	δ_9	δ_{10}	δ_{11}	δ_{12}	δ_{13}
	-0.6141	-0.8796	-0.8747	-0.8036	-0.8614	-0.8276
	δ_{14}	δ_{15}	δ_{16}	δ_{17}		
	-0.8795	-0.9026	-0.9342	-0.8836		
	V_4	V_5	V_6	V_7	V_8	V_9
	-0.5932	**-1.0238**	-0.5400	-0.6248	-0.6933	-0.3131
	V_{10}	V_{11}	V_{12}	V_{13}	V_{15}	V_{16}
	-0.2358	-0.3742	-0.2594	-0.3804	-0.1823	-0.2188
	V_{17}	λ				
	-0.2012	**1.0000**]^T				

T = transpose

dV_5 = -1.0238; from Fig. 5: -1.020. The differences in the values are a consequence of the limited numerical accuracy.

Iterations 15,17 and 19 with voltage V as continuation parameter.

The tangent vector components accounted for by the CPF in the iteration 15 are:

$t_{15}=$	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7
	[-0.7472	-0.7510	-0.8082	-1.4047	-0.8069	-0.5251
	δ_8	δ_9	δ_{10}	δ_{11}	δ_{12}	δ_{13}
	-0.5937	-0.8493	-0.8419	-0.7749	-0.8292	-0.7993
	δ_{14}	δ_{15}	δ_{16}	δ_{17}		
	-0.8463	-0.8667	-0.8925	-0.8494		
	V_4	V_5	V_6	V_7	V_8	V_9
	-0.5789	**-1.0000**	-0.5267	-0.6098	-0.6764	-0.3025
	V_{10}	V_{11}	V_{12}	V_{13}	V_{15}	V_{16}
	-0.2274	-0.3632	-0.2504	-0.3684	-0.1725	-0.2035
	V_{17}	λ				
	-0.1919	**0.6133**]^T				

T = transpose

The tangent direction for the continuation parameter V_5 results from the values taken from t15 to $d\lambda$ / dV_5 = -0.6133 / 1= -0.6133. From Fig. 6 with -0.610 again we get a good match. The same applies to the iterations 17 and 19, see tangent vectors below.

Tangent vector in the iteration 17:

$t_{17}=$	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7
	[-0.7800	-0.7756	-0.8112	-1.4159	-0.8022	-0.5063
	δ_8	δ_9	δ_{10}	δ_{11}	δ_{12}	δ_{13}
	-0.5900	-0.8267	-0.8139	-0.7517	-0.8017	-0.7781
	δ_{14}	δ_{15}	δ_{16}	δ_{17}		
	-0.8178	-0.8340	-0.8498	-0.8192		
	V_4	V_5	V_6	V_7	V_8	V_9
	-0.5779	**-1.0000**	-0.5249	-0.6079	-0.6743	-0.2957
	V_{10}	V_{11}	V_{12}	V_{13}	V_{15}	V_{16}
	-0.2214	-0.3581	-0.2442	-0.3617	-0.1615	-0.1832
	V_{17}	λ				
	-0.1825	**0.0851**]^T				

T = transpose

Tangent vector in the iteration 19:

$t_{19}=$	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7
	[-0.8228	-0.8050	-0.8091	-1.4284	-0.7875	-0.4723
	δ_8	δ_9	δ_{10}	δ_{11}	δ_{12}	δ_{13}
	-0.5726	-0.7841	-0.7636	-0.7090	-0.7521	-0.7382
	δ_{14}	δ_{15}	δ_{16}	δ_{17}		
	-0.7665	-0.7763	-0.7770	-0.7655		
	V_4	V_5	V_6	V_7	V_8	V_9
	-0.5757	**-1.0000**	-0.5216	-0.6032	-0.6700	-0.2851
	V_{10}	V_{11}	V_{12}	V_{13}	V_{15}	V_{16}
	-0.2120	-0.3494	-0.2342	-0.3510	-0.1449	-0.1527
	V_{17}	λ				
	-0.1682	**-1.0975**]^T				

T = transpose

Iterations 20, 25 and 32 with continuation parameter lambda

Similar calculations can also be carried out for the lower part of the PV curve with the continuation parameter lambda, which is not done here. The tangent vector components of the iterations 20, 25 and 32 given below are used to develop **Fig. 7.**

Tangent vector component in the iteration 20:

$t_{20}=$	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7
	[-0.5308	-0.5152	-0.5080	-0.9044	-0.4903	-0.2857
	δ_8	δ_9	δ_{10}	δ_{11}	δ_{12}	δ_{13}
	-0.3542	-0.4791	-0.4637	-0.4316	-0.4567	-0.4510
	δ_{14}	δ_{15}	δ_{16}	δ_{17}		
	-0.4652	-0.4694	-0.4652	-0.4638		
	V_4	V_5	V_6	V_7	V_8	V_9
	-0.3626	**-0.6314**	-0.3281	-0.3788	-0.4212	-0.1766
	V_{10}	V_{11}	V_{12}	V_{13}	V_{15}	V_{16}
	-0.1308	-0.2176	-0.1446	-0.2180	-0.0863	-0.0869
	V_{17}	λ				
	-0.1017	**-1.0000**]^T				

T = transpose

In the Iteration 25:

t_{25}=

δ_2	δ_3	δ_4	δ_5	δ_6	δ_7
[-0.1583	-0.1399	-0.1148	-0.2506	-0.0972	-0.0321
δ_8	δ_9	δ_{10}	δ_{11}	δ_{12}	δ_{13}
-0.0635	-0.0695	-0.0591	-0.0575	-0.0580	-0.0649
δ_{14}	δ_{15}	δ_{16}	δ_{17}		
-0.0588	-0.0550	-0.0424	-0.0566		
V_4	V_5	V_6	V_7	V_8	V_9
-0.1006	**-0.1872**	-0.0893	-0.0984	-0.1137	-0.0412
V_{10}	V_{11}	V_{12}	V_{13}	V_{15}	V_{16}
-0.0286	-0.0519	-0.0317	-0.0518	-0.0117	-0.0016
V_{17}	λ				
-0.0174	**-1.0000**]^T				

T = transpose

In the Iteration 32:

t_{32}=

δ_2	δ_3	δ_4	δ_5	δ_6	δ_7
[-0.0272	-0.0011	0.0340	-0.0627	0.0570	0.0569
δ_8	δ_9	δ_{10}	δ_{11}	δ_{12}	δ_{13}
0.0294	0.0883	0.0947	0.0846	0.0927	0.0809
δ_{14}	δ_{15}	δ_{16}	δ_{17}		
0.0951	0.1017	0.1137	0.0971		
V_4	V_5	V_6	V_7	V_8	V_9
-0.0287	**-0.0759**	-0.0246	-0.0243	-0.0300	-0.0060
V_{10}	V_{11}	V_{12}	V_{13}	V_{15}	V_{16}
-0.0021	-0.0088	-0.0024	-0.0084	-0.0061	-0.0170
V_{17}	λ				
-0.0031	**-1.0000**]^T				

T = transpose

3.2. COURSE OF THE QUOTIENT OF THE LARGEST (NEGATIVE) VOLTAGE TANGENT VECTOR COMPONENT AND LAMBDA TANGENT VECTOR COMPONENT

Fig. 7 shows the quotient between the largest negative voltage tangent vector component and the lambda tangent vector component. The scale for this quotient can be found on the ordinate of the lower part of the figure. Since it also defines the negative slope of the tangent (see Fig. 3), the figure also contains an angle scaling, namely on the abscissa of the lower part.

The upper part of the figure shows the PV curve, with the abscissa being the voltage axis and the ordinate being the lambda axis. The V-axis scaling is non-linear, so the PV curve is mapped symmetrically, putting the critical point right in the middle. (In fact, the PV curve is usually asymmetrical with respect to the critical point). The values for the development of the Fig. 6 are taken from subchapters 3.1.1. and 3.1.2.

The purple curve on the right describes CPF iterations 1 to 13. The middle blue curve applies to iterations 14-19. And, the left purple curve represents iteration 20 – 32, (see subchapter 3.1.2).

The starting point is on the right side of the Fig. 7. The continuation parameter is the load factor λ. The quotient $dV/d\lambda$ and the tangent inclination are still small for the time being. In particular, $d\lambda$ = +1; dV < 0, $|dV|$ < 1). The increase in load (λ increases) results in the increase of the absolute value of the quotient according to a tangent function, see purple curve. At point S it takes the value -1 ($d\lambda$ = +1, dV = -1).

This corresponds to a tangent inclination of tg^{-1}(-1) = -45°. If the load factor is further increased, the voltage tangent vector component becomes larger than the lambda tangent vector component (point E), and therefore continuation parameter changes from λ to V.

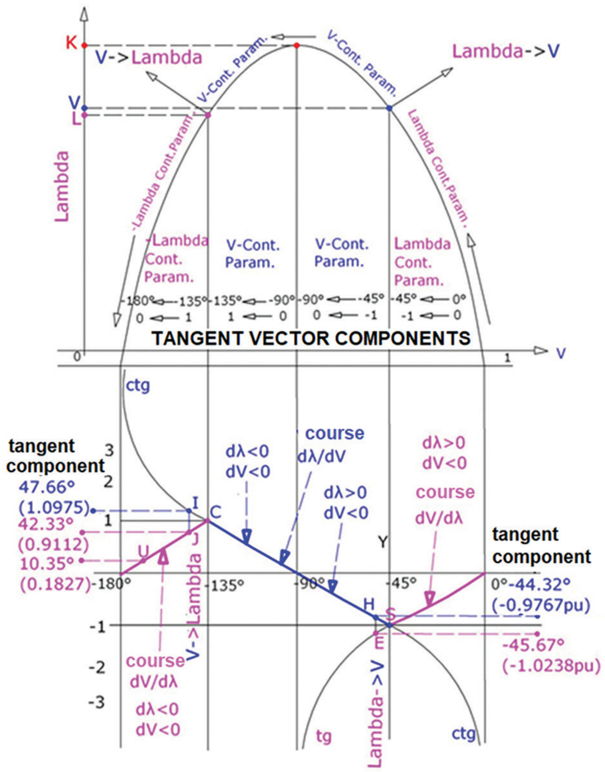

Fig. 7. The component of the tangent vector of the higher (negative) voltage compared to the lambda tangent vector component, values for the 17- node network.

In order for the quotient of the tangent vector components to remain finite, ($d\lambda/dV$) needs to get inverted. Therefore, its further course develops according to a cotangent function (blue curve). The tangent angle of -45.67° thus corresponds to a quotient of -1/1.0238 = -0.9768 (see point H on the blue curve). This is now valid for all further CPF iterations carried out with Continuation Parameter V. The critical point is reached at -90 (corresponding to ctg(-90) = 0 or $dV = \infty$).

At a tangent inclination of -135°, both $d\lambda$ and dV have the value -1, and the quotient of the tangent vector components becomes 1, see point C. For the development of point I, see Chap. 3.1.2 for the iteration 19.: dV = -1, $d\lambda$ = -1.0975,

$$ctg\text{-}1(d\lambda/dV) = ctg\text{-}1(1{,}0975/1) = 42{,}33°.$$

However, since the tangent slope has exceeded the critical value of -90°, a 180° transformation must be performed, which is why the final angle is: 42.33° - 180° = -137.66°. The condition for resetting the continuation parameter from V to λ is now fulfilled ($d\lambda > dV$). Accordingly, the quotient $d\lambda/dV$ is inverted to $dV/d\lambda$, and the purple tangent function shown on the left in Fig. 7 is valid for the rest of the curve.

Point I on the blue curve now corresponds to point J on the purple curve.

($tg^{-1}(d\lambda/dV) = tg^{-1}(1/1{,}0975) = tg^{-1}(0{,}9112) = -137{,}66°$).

For the development of a further point of the left purple curve, values from Chap. 3.3 are taken, for instance, for the iteration 25:

$d\lambda$=-1, dV= -0.1827; tg^{-1}(dV/dλ) = tg^{-1}(0,1827) = 10,35°. After transformation by 180°, the final tangent inclination is 10.35 – 180 = -169.65°. In the figure, this corresponds to point U.

4. CONCLUSIONS

This Paper illustrates the voltage stability by using the slope of the tangent vector component. It represents a research and detailed explanation of Lambda as a continuous parameter. The tangent angle was calculated from the quotient between the voltage and the largest lambda tangent vector component. Moreover, it illustrates the Voltage V as continuous parameter, where the tangent angle was calculated from the coefficient between lambda λ and the largest tangent vector component of the voltage. Furthermore, it compares the tangent vectors when calculated with the continuous power flow program and when directly calculated, for the most vulnerable network node. The highest (negative) voltage tangent vector component occurs in the node 5 in the network with 17 nodes. The results for the tangent vectors calculated through the CPF program were compared with the results directly calculated with mathematical formula and both results match. The chart, which contains the classical PV curve and the flow of tangent vectors during the CPF iterations, was developed based on the results obtained. In addition to the classic Power-Voltage (PV) curve, this serves as an additional indicator for ensuring voltage stability of the examined system. This enables easier comparison with the effectiveness of different measures regarding the improvement of the voltage stability of the examined system.

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