A Study on A Novel Collision Risk Prediction Map for Maritime Traffic Surveillance Based on Ship Domain

Original Scientific Paper

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Abstract *– Recently, a regional model for assessing the risk of multi-ship collision has been developed to reduce the risk of ship collision in territorial sea areas such as trade ports and entry waterways and improve the safety and efficiency of ship traffic. The focus is on marine traffic in the visualized waters with the risk of ship collision. However, due to the lack of information from experts with sufficient knowledge and experience in a given area, they also have some limitations in adequately and comprehensively representing the risk of collision, especially in busy waterways where encounters of more than two ships often appear. In addition, they could not visualize the location of the proximity collision and the exact risk value in real time. Therefore, to overcome the limitations of previous studies, this paper proposes a new regional collision risk visualization system, which combines density-based spatial clustering of applications with noise (DBSCAN) and analysis and knowledge-based ship domains and uses AIS data to intuitively and accurately map the dynamic collision risk of water areas at successive moments, predict areas where collisions can happen by dynamic risk index and warn the ships. Identifying high-risk collision areas between multi-ships can be enhanced using the developed system, which allows for reliable and accurate analysis to help implement safety measures.*

Keywords: *ship collision, collision risk, maritime traffic, heat map, ship domain*

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1. INTRODUCTION

Frequent encounters between multiple ships in congested waters are one of the main factors causing ship collisions. However, there are significant challenges to understanding complex multi-ship encounter situations. Most accidents are caused by human error, which cannot be prevented as long as people operate the ship. Collisions with ships at sea can cause severe loss of life and property, and some may even seriously impact the marine ecological environment. Therefore, detecting and judging collision risk is the primary task of intelligent navigation of ships. The increased ship traffic at a given moment can complicate ship traffic, make congested waters more congested, and increase the likelihood of ship collisions.

Multi-ship encounters at sea are highly complex and uncertain, which can be a big challenge for ships. A quick and correct grasp of the current situation is needed to make appropriate maneuvering decisions and perform well in multi-ship encounters. Most recent frameworks need to provide a general picture of a complex problem. Influencing factors such as speed, heading, environment, and maneuverability should be considered. The size-related factor that affects the encounter, such as the ship's length, should also be considered. The larger the size, the higher the risk of collisions between ships at similar distances.

It is necessary to evaluate the potential danger between ships in real time [1] and express the collision risk in the chart, which can also provide a better reference for collision avoidance operations [2]. Many researchers have recognized that it is helpful for navigators to be vigilant about real-time collision risks, facilitating decision-making. A correct and complete understanding of complex multi-ship encounters is an essential means and premise to prevent ship collisions and ensure the safety of ship navigation.

To solve the above problems, this study proposes a new framework for early detection of collision risk between ships in congested waters to notify the Officer on Watch (OOW) or Vessel Traffic Service Officer (VTSO) of potential collisions so that the person in charge can make decisions by observation. Considering the area's complexity, a new ship domain (SD) concept will determine the near collision. Near collision is the situation when SDs overlap. Overlapping areas can be displayed as indexes. In addition, a method of using an index to display the areas where the ship may collide with other ships is proposed, which is presented in the form of a collision heat map so that OOWs and VTSOs can quickly identify the danger areas where collisions may occur in congested waters. As a result, risk awareness has also increased. The main contributions of our framework are:

- Propose Heat Ship Domain can calculate continuous and dynamic potential collision risk levels around a ship in a maritime waterway. It considers the ship's static and dynamic characteristics and experts' knowledge of particular water.
- Propose a Dynamic Collision Risk Index that calculates the impact of multiple ships on an area and detects high-value index areas by using the overlap function of the Heat Ship Domain.
- Propose a Collision Risk Prediction Map that assesses a waterway's regional risk and identifies and displays high-risk hotspots based on the Dynamic Collision Risk Index. This map would allow OOWs and VTSOs to make evasive decisions for collision avoidance by observing it.

The remainder of the paper is organized as follows: Section 2 provides a literature review of work related to collision risk alert systems, focusing on their merits and demerits. Section 3 discusses the methodology, including techniques and algorithms for constructing a new SD based on the Kernel Density Function and using this SD to establish a collision heat map that displays collision risk areas in congested waters. Section 4 presents a case study of multi-ship encounter scenarios to validate the proposed methodology. Finally, Section 5 presents conclusions about the results.

2. RELATED WORK

Ship collision risk identification is attractive, particularly in specific water areas. To improve the safety of navigation, many scholars have been studying solutions to visualize the geographical distribution of collision risk. To achieve this goal, the risk of collision between ships should first be accurately quantified. In various models and navigation practices, the distance to the closest approach point (DCPA) and the time to the closest approach point (TCPA) are the essential criteria for "collision risk" and the most critical parameters [3]. However, DCPA and TCPA are only used to show the risk of collision based on subjective judgment and cannot generate applicable quantitative values for collision risk.

Furthermore, DCPA and TCPA are hard to apply in busy waterways with high ship density. Therefore, another index, named the Collision Risk Index (CRI), was introduced, which evaluates the probability of a collision [4]. Fuzzy logic has been used in various collision risk assessment and collision avoidance decision-making methods [5, 6]. However, geometric information needs to be considered, and the risk of collision with multiple ships is not discussed. The velocity obstacles-based framework assesses the risk of colliding velocities [7, 8]. Although this research has effectively calculated collision risk, they can only apply in open sea areas. In congested waterways with smaller room for ship maneuvering and complex conditions, the collision risk could be more comprehensively achieved, especially when there are more than two ships.

Still, it is difficult to assess the risk of a collision given the geographical conditions. Although these methods can determine collision risk based on encounter conditions and ship maneuvering, they need evaluation results that combine geometric information. Therefore, the concept of "ship domain" is used to find available maneuvering space and geometric information can be used to assess collision risk. The ship domain was first defined as "the domain around a ship underway which most navigators of following ships would avoid entering" [9] and "the effective area around a ship which a navigator would like to keep free to other ships and stationary objects" [10]. The ship domain model is a quantitative tool for assessing the risk of collision when another ship intrudes into the range of another ship and has been widely used for different purposes, such as ship collision avoidance [11, 12], near misses, and hotspot identification [13, 14]. In the above models, ship domains are assumed deterministic, and the domain parameters have not been extended. If a target is outside the boundary, it is safe; no action needs to be taken; if the target is inside the boundary, it is dangerous, and action must be taken to keep it out of the boundary. The collision risk is only 0 and 1; the level of risk can not be exactly presented. There are some advantages and disadvantages to ship domain model applications. Subjectivity is a problem in knowledge-based and analysis-based ship domain models, as the models rely heavily on the judgment of navigators, experts, and researchers—the human factor is not considered in the empirical ship domain model [15]. The human factor is crucial because when a ship has an accident, the leading causes are human error, which is caused by insufficient knowledge of the operation, receiving wrong or inadequate information to make judgments, or unfamiliarity with the environmental characteristics of the water area [16]. A framework is needed to formulate a mathematical model that considers ship size, speed, and a human factor component.

To improve safety, several studies have developed a framework for assessing regional collision risk, which combines density complexity and multi-shi collision risk. The risk of a collision off the coast of Portugal is evaluated by predicting future distances between ships based on AIS data. This approach can be only used to identify collision candidates in complex traffic patterns in the long term but not in real-time [17]. Maritime traffic around the Shetland Islands is visualized in the form of AIS pings maps, ship density maps, ship trajectory maps, ship length maps, etc., to ensure the safety of navigation in marine space and development planning [18]. The molecular collision theory establishes an encounter probability map of the Istanbul Strait [19]. After summarizing the risks through the radial distribution function, the spatial interpolation technique was used to identify the geographical distribution of the collision risks in the Bohai Strait [20]. While these methods effectively visualize maritime traffic in waters, some things could be improved. The spatial distribution of the encounter probability within these models is calculated for large areas. This means that the whole area will have the same index value.

A kinematics feature-based vessel conflict ranking operator is introduced to evaluate ship collision risk by integrating the relative position vector and the relative velocity, accounting for static and dynamic information of AIS to quantify ship collision risk and identify high collision risk areas. However, this paper needs to consider the impact of multi-ship, which is only available in open-sea regions [21]. Another method for identifying ship navigation risks is combining the ship domain with AIS data to increase collision risk identification prediction accuracy for ship navigation in complex waterways. This method constructs a ship domain model based on the ship density map drawn using AIS data. Then, the collision time with the target ship is calculated based on the collision hazard detection line and safety distance boundary, forming a method for dividing the danger level of the ship navigation situation. The risk level is only evaluated when the target ship is inside the outside ship domain and the intersection of the boundary [22]. Fuzzy logic calculates the collision avoidance maneuver for the selected ship, considering the closest point of approach, relative bearing, and the ship's speed. Evaluate the collision risk and navigation situation based on COLREG rules, sort the target vessels, and determine the most dangerous vessel. Multi-ship encounters are considered but only in the vast open sea [23]. An anchorage collision risk model was established in microscopic, macroscopic, and complexity aspects, which considered ship relative motion, anchorage characteristics, and ship traffic complexity. In modeling complexity, it would be better to incorporate the factors of ship motion to make the consideration of traffic complexity more sufficient [24]. A dynamic elliptical ship domain based on AIS data combines the relative motion between ships in different encounter situations to assess the level of ship intrusion in the domain. However, during the movement, the size of the ship domain is static, not changing with speed [25].

Furthermore, these studies have been used for maritime traffic analysis, and the risk index used for the location of future collisions needs to be taken into account. It is difficult to distinguish the collision risk between collision candidates and obtain an accurate collision risk value. In addition, identifying appropriate potential collision areas in congested waters is challenging. Still, no standard scale for measuring the degree of collision criticality exists. Therefore, adopting specific criteria to determine the collision risk and warn the OOWs and VTSOs is critical.

Based on these circumstances, this study developed a clustering-based regional collision risk prediction model for a collision area using a new ship domain and considering geographical patterns. The method aims to detect collision risk quickly and dynamically using AIS data from short intervals. The establishment of our model is presented in Section 3.

3. PROPOSED METHOD

Collisions in highly complex maritime traffic pose significant risks. In high-density waters, it is expected to encounter a group of ships. If the complexity exceeds the threshold, the likelihood of a near collision rises significantly. In crowded waters, the ship risks colliding with multiple target ships. The degree of collision between multiple ships needs to be integrated. The degree of danger increases with the number of target ships at risk of collision. In this section, the methodology of real-time collision risk assessment indicators using the ship domain is developed as the basis for the safe navigation of ships. The diagram of the proposed framework is shown in Fig. 1.

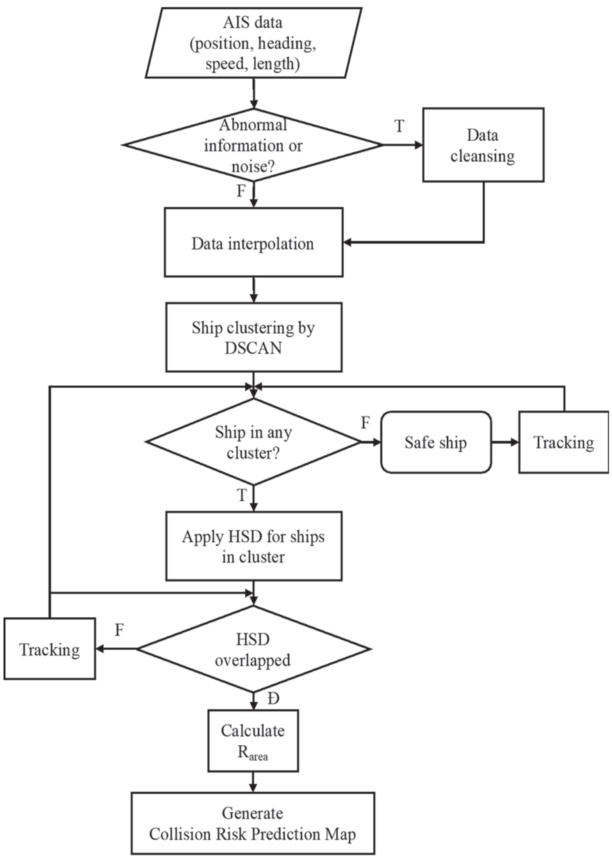


Fig. 1. Diagram of proposed framework

3.1. DATA PROCESS

AIS data is increasingly being used as a valuable source of information on ship traffic in maritime traffic engineering and maritime transport safety studies. The AIS identifies each ship equipped with an AIS transmitter and transmits static information about the ship (call sign, IMO number, destination, cargo, etc.) and frequent updates about the ship's position, speed, and heading [26]. The AIS data is decoded by extracting position, speed, heading, size, time, and MMSI information.

The main task of this step is to make the AIS data reliably used for calculation. More specifically, AIS data should be updated and interpolated correctly over time intervals.

Abnormal information or noise can significantly affect the regional ship collision risk assessment. Therefore, given the integrity of the real-time data, it is not appropriate to delete those noisy records, which should be cleaned and updated. A 4-step process was used for the data cleansing step in this study [12]. According to Newton's laws of motion, the average speed can be calculated as the ratio of the distance traveled to the travel time. Therefore, the ship's position recording and acceleration and deceleration capabilities can be used to check whether the speed record is within a reasonable range. Correspondingly, the updated speed data can be used to clean the location data based on the same principle.

Step 1: Check the reasonableness of the speed data.

Step 2: Update the irrational speed data.

Step 3: Check the reasonableness of position data.

Step 4: Update the position data.

In addition, AIS data is sent randomly at different times. To prevent the temporal dispersion of AIS data, the updated AIS data is interpolated every 30 seconds to obtain information simultaneously [27]. Calculate the movement of each ship and estimate the position in 30 seconds. After extensive cleansing and pre-processing, the original AIS database becomes a suitable dataset for analysis.

3.2. SHIP DOMAIN CONSTRUCTION

Given the complexity, this subsection intends to construct a new ship domain to identify a potential collision that is defined as one that occurs when the ship domains of the local ship (OS) and the target ship (TS) overlap, as shown in Fig. 2.

The concept of ship domain is reflected in COLREG, where ships must pass through each other and obstacles at a safe distance. This safety distance represents the domain of the ship. Several ship domains have been proposed to express the hazard level within the domain [28-30]. However, choosing the size and shape of the ship domain best suited for ship navigation takes a lot of work, especially in congested waters. Most of the existing collision risk identification methods are based on a geometric perspective and use indicators such as distance to measure collision risk, which requires more expert knowledge.

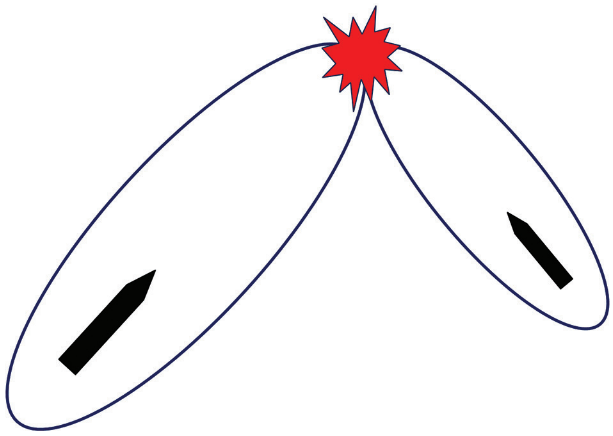


Fig. 2. Ship domains overlap

The ship's domain is essential for classifying according to the severity of the encounter since the encroachment on the domain implies a certain level of proximity that the navigator usually wishes to avoid. In addition, other contextual characteristics should be considered. In meetings between ships, larger ships have larger domains. Each ship has its turning circle. The turning circle of a ship determines the ease and rapidness with which a ship can change its course or direction. The greater the size, the larger the turning circle. Larger ships need more space for maneuvering than smaller ones. Some authors use geographic information technology and AIS data to calculate how the space around ships is used (or kept free) during their movements. They have found that the ship's size affects its domain, and smaller ships tend to meet slightly closer than larger ones. This means that safety areas increase along with the size of the ship. One can notice that the bigger the ship, the bigger the domain becomes. This means that for larger ships in a given encounter, the situation may be classified as dangerous for larger ships but not for smaller ships, where the situation may still be considered safe. This problem can be solved using a new ship domain using field theory.

The concept of "field" is abstracted as a mathematical concept used to describe the distribution of a particular physical quantity or mathematical function in space. There are both connections and differences between the various fields. Fields can be expressed abstractly with mathematical models. Any object can form a field, and different objects produce different fields. The field theory-based method has been widely used in vehicle safety research, but there are relatively few research results in the safe navigation of ships [31]. Due to the differences in traffic characteristics between navigation areas, using one type of ship domain for each area is difficult. Therefore, inspired by field theory, a new ship domain is introduced to measure the degree of collision risk around ships. The new ship domain applies to regions and the probability levels of advanced decision-making systems better suited for navigational risk detection.

The coordinates of the ship encounter are shown in Fig. 3. The origin represents the own ship (OS). It is located in the OS's center, and the OS's velocity vector relative to the target ship constitutes the Y-axis through the origin. The X axis is perpendicular to the Y axis and passes through the origin. The line of motion of the target ship (TS) relative to the OS is parallel to the Y axis. Assuming that the TS is located at point P with coordinates (x_p, y_p), then point A is the projection of P on the X-axis. The PA line is the relative line of motion of the target ship at P relative to the OS, and the projection point A is the CPA of the target ship to the OS, i.e., the distance (d_{OA}) from the origin O to point A is the DCPA from the operating system to the target ship. The distance from point P to point A (d_{PA}) is from the target ship to the CPA, which is the product of the time required to reach TCPA and the speed V [32]. Moreover, assume that except for real target ship P, there are a lot of imaginary target ships P_n with no speed. The non-dimensional of d_{OA} and d_{PA} are calculated as follows:

$$d'_{OA} = \frac{DCPA_p}{L} = \frac{|x_p|}{L} \tag{1}$$

$$d'_{PA} = \frac{TCPA_p \times V}{L} = \frac{|y_p|}{L} \tag{2}$$

where
d'_{OA} is non-dimensional of dOA;
d'_{PA} is non-dimensional of dPA;
V is the speed of the ship;
L is the length of the ship;

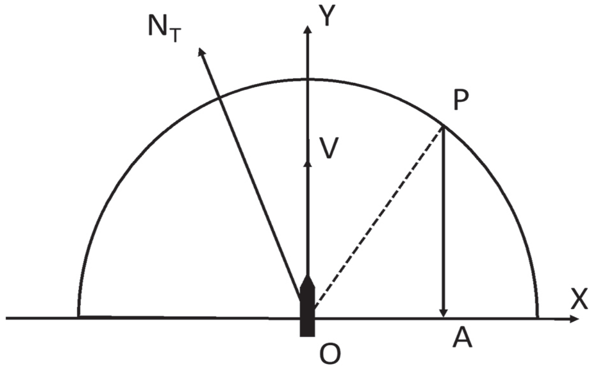


Fig. 3. The coordinates of the ship encounter

Kernel density estimation (KDE) generates a smoothed empirical probability density function based on individual locations across all sample data. This estimate better represents the "true" probability density function of a continuous variable [33].

The radial kernel estimator is based on the Euclidean distance between an arbitrary point $\{x,y\}$ and sample point $\{x_i, y_i\}$, $i = 1,2, ..., n$:

$$\hat{f}(x,y) = \frac{1}{nh_x h_y}\sum_{i=1}^{n} K(\sqrt{\left(\frac{x_i - x}{h_x} + \frac{y_i - y}{h_y}\right)^2}) \tag{3}$$

where
n is the number of sample points;
K is the kernel function;
h_x, h_y are the smoothing parameters in the X-axis and Y-axis.

KDE is applied to establish a new dynamic collision risk (DCR) around OS, employed d'_{OA} and d'_{PA}. DCR value is calculated for every point around a ship:

$$DCR(P) = \frac{1}{nh_x h_y}\sum_{i=1}^{n} K(\sqrt{\left(\frac{d'_{OA}}{h_x} + \frac{d'_{PA}}{h_y}\right)^2}) \tag{4}$$

Assume that the smoothing parameter in X-axis and Y-axis have the same value ($h_x = h_y = h$). After applying the asymmetric Gaussian function as a kernel function, the DCR of a point P can be expressed as follows:

$$DCR(P) = \frac{1}{nh^2}\sum_{i=1}^{n} \frac{1}{\sqrt{2\pi}} e^{-\frac{(x_p+y_p)^2}{2L^2h^2}} \tag{5}$$

where
$DCR(P)$ is the Dynamic Collision Risk Index of point P;
h is the influencing parameter, which represents water areas and is named the area parameter.

One ship will have different sizes of ship domains in different navigation areas.

DCR is proposed to analyze and measure the collision-risk degree of every point around OS, including real and imaginary target ships, considering DCPA, TCPA, ship length, and speed.

The risk index at a point is related to the coordinate value at this point. Every point in the vicinity of the ships has a value of DCR, and points with the same value of DCR will be shown as contour lines or the same color to indicate the same degree of risk. Fig. 4 shows the visualization of DCR in two forms: contour lines or colors of DCR. DCR can be depicted as an elliptical area surrounding the ship, consisting of multiple levels of risk. This area is called the Heat Ship Domain (HSD). Each level within the HSD is represented by a color or isotherm that connects the points with the same value in the field to make the iso risk index line. The fundamental concept is that when the ships move closer, their respective ship domains will overlap. The overlapped area, which can be understood as the potential collision area, is the collision position. This area will vary at different moments, and the changes in the overlapped area also demonstrate the degree of collision probability at the moment of encounter. In case of a multi-ship encounter, at each moment when ships are approaching each other, values of DCR of points between these ships increase according to their positions to ships. The influence of these ships in this area is higher than in other areas. Due to the change of DCR, the color describes the area with a probability of collision accident changing from cool to hot. The DCR is a cost-like value. It tends to be higher for the higher of the collision risk. The points with high values of DCR indicate that there will probably be a collision there if the ships involved do not perform the evasive action and the magnitude of the action required to clear the situation.

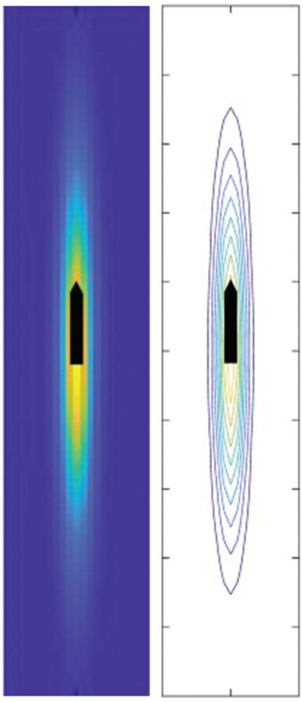


Fig. 4. Appearances of Heat Ship Domain

The speed and heading of the ship determine the extension and direction of the longitudinal axis of HSD. The area parameter also specifies the coverage of HSD. The next step is to determine the edge of HSD, corresponding to iso risk line 0,1 (DCR = 0,1).

Many factors affect the size of the ship domain, but only a few can be considered in the domain size determination process for practical reasons. The first is the human factor, which includes navigators' skills, knowledge, and mental and physical abilities [34]. Also, according to experts, another critical factor is the type of water used [35].

Four points are analyzed for the boundary of HSD to reveal the ship's passing distance. The domain proposed in this paper considers ship speed, ship length, and area parameter h. For a ship of a specific size and speed, different h will lead to varying edges of HSD, corresponding to iso risk line 0,1 (DCR = 0,1) (as shown in Fig. 5). The value of h depends on navigators and the characteristics of the water area and can be determined by expert knowledge methods.

The navigator's knowledge of the assessment of the navigation situation provides the basis for determining the safe distance between ships in one particular area with lengths overall, respectively: under 115m, 116 – 145m, 146 – 175m, and over 175m m. Suppose ships are traveling at a speed of 8 knots. An expert study was conducted to assess ships' encounters in Haiphong Port waters in conditions of good visibility. The participants were navigators, including captains, OOWs, sea pilots, and VTSOs with different sea experiences. The study took the form of a questionnaire.

Respondents were required to give answers about safety distances in four directions: fore (a), aft (b), port (c), and starboard (d) from OS with four ship lengths, as mentioned in Fig. 6.

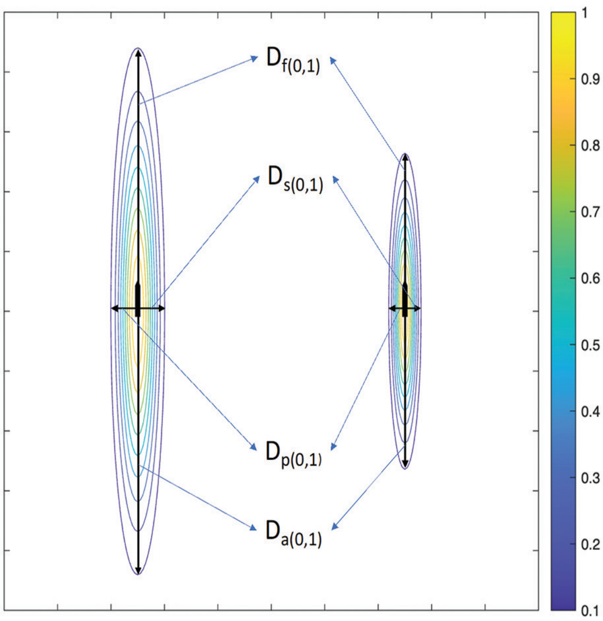


Fig. 5. Size of HSD changes with area parameter

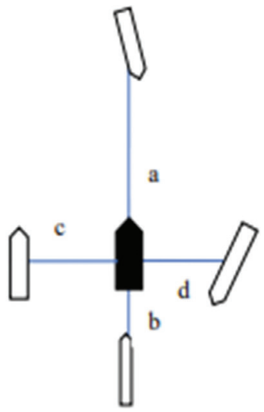


Fig. 6. Safe distances around a ship

The survey was conducted over three months and had more than 300 participants. The results of the survey are shown in Table 1-4. With each type of ship, the safe distances from OS to four directions are collected as minimum, maximum, and mean values. The pilots have a lot of experience navigating in the study area, so they believe the safe distance between ships in front of the ship can be manageable. On the contrary, a more considerable distance is required for the officers, especially the 3rd officer, to ensure no collision risk. According to experts, the dangerous distance on both sides has relatively uniform results. Due to the characteristics of Haiphong Ports water – narrow width (average 80m) and depth, it is unavailable for the long passing distance from starboard and port sides. It can be seen that, in this area, experts all believe that the collision avoidance distance on both sides is similar. Some empirical studies that used the AIS data in narrow channels or restricted areas to measure the space around a ship during navigation have shown different results in the size of ship domains. However, the dangerous distances on both sides of the ship are similar.

Table 1. Safe distances of ships under 115m

	Minimum (m)	Maximum (m)	Mean (m)
Fore (a)	350	900	580
Aft (b)	350	900	520
Port (c)	15	40	20
Starboard (d)	15	40	25

Table 2. Safe distances of ship 116 – 145m

	Minimum (m)	Maximum (m)	Mean (m)
Fore (a)	400	900	670
Aft (b)	400	900	560
Port (c)	15	40	22
Starboard (d)	15	40	25

Table 3. Safe distances of ship 146 – 175m

	Minimum (m)	Maximum (m)	Mean (m)
Fore (a)	450	1300	750
Aft (b)	450	1300	650
Port (c)	20	40	28
Starboard (d)	20	40	30

Table 4. Safe distances of vessels over 175m

	Minimum (m)	Maximum (m)	Mean (m)
Fore (a)	600	1600	850
Aft (b)	600	1600	780
Port (c)	20	40	32
Starboard (d)	40	40	35

These survey results will be used as subject data for the approximation process to determine area parameter h. As can be seen, a particular area will have a value of h. For this aim, the Least Squares method is used to find very close or the same solution as the optimal values [36]. The boundary of the iso risk index line 0,1 of HSD will be differently generated by each value of h and compared with the collected data. The best-fit boundary takes the minimum value of the fitness function and returns the optimal value of h. The fitness function is as follows:

$$\begin{cases} \Delta d_f(h) = (D_{f(0,1)}(h) - a)^2 \\ \Delta d_a(h) = (D_{a(0,1)}(h) - b)^2 \\ \Delta d_s(h) = (D_{s(0,1)}(h) - c)^2 \\ \Delta d_p(h) = (D_{p(0,1)}(h) - d)^2 \end{cases} \tag{6}$$

$$\Delta d(h) = \sum_{i=1}^{4} \Delta d_i(h) \tag{7}$$

where

$D_{f(0,1)}$, $D_{a(0,1)}(h)$, $D_{s(0,1)}(h)$, $D_{p(0,1)}(h)$ are the radii of iso risk index line 0,1 of HSD in fore, aft, starboard, and port side, respectively;

$\Delta d_f(h)$, $\Delta d_a(h)$, $\Delta d_s(h)$, $\Delta d_p(h)$ are squared distance differences at fore, aft, starboard, and port side, respectively;

$\Delta d(h)$ is the sum of squared distance differences.

The fitness function of the approximation for the size of iso risk index line 0,1 of HSD in the study area involving area parameter h is calculated by Equation 7. The parameter value of h can be estimated using the sum of squared distance differences between radii in the proposed model and the surveyed data. It means that after finding the minimum value of the fitness function ($min\Delta d$), the corresponding parameter can be selected as in Table 5.

Table 5. Area parameter h for different ship lengths

Ship length	Under 115 m	116-145 m	146-175 m	Over 175 m
h	0.22	0.21	0.2	0.2

The motivation of this study is to utilize the overlap between ship domains to identify potential collision risks and visualize this area. A questionnaire about the visualization of HSD was carried out with VTSOs in Haiphong Ports water. They realized that if the elliptical-shaped domains are used, it will lead to a misunderstanding of the collision situation for VTSOs and confuse them in cases where a ship was following or crossing aft of another, the forward part of one ship domain overlaps the aft part of another ship's domain (as shown in Fig. 7).

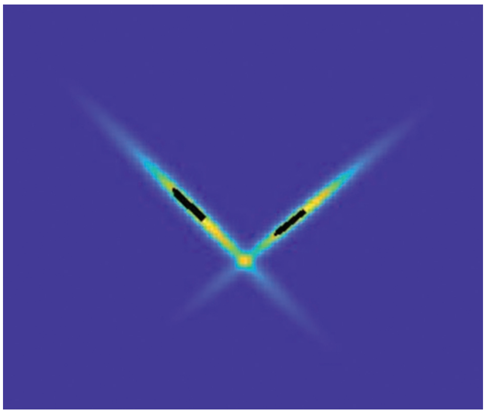


Fig. 7. False potential collision area

Although an overlapped area happens, there is no collision risk in this case. Therefore, the aft part of HSD will be shortened to the value of the starboard and port sides, according to the questionnaire with VTSOs, as shown in Fig. 8, and samples of HSD with speed 8kn in Haiphong Port water are presented in Fig. 9. Different ship lengths will have different sizes of HSDs.

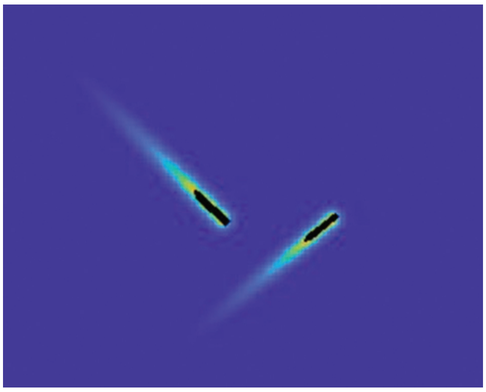


Fig. 8. Shorten HSDs

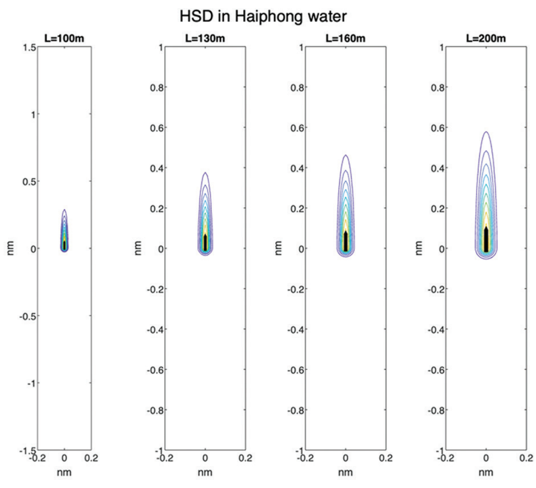


Fig. 9. HSDs of ships with different lengths in Haiphong water

The resulting past ship domain with speed 10kn in restricted areas is reproduced in Fig. 10 along with the edge of HSD, corresponding to iso risk line 0,1. The comparison shows that the ship domain of Coldwell [37] is reasonably compatible with the proposed ship domain on the forward but overestimates the space requirement on the starboard and port sides. Other ship domains have shorter fore parts, but three other sides appear overestimated. The reason is that HSD is constructed based on the characteristics of the Haiphong waterway based on experts' experiences. It should be noted that the Fujii et al. [9] and Hansen et al. [11] models still cannot take into account the effect of change in speed, while Wang and Chin's model does not enlarge significantly with increasing speed [38]. With the more extended fore parts and changing during navigation, HSD is suitable for early detection risk, especially when crossing narrow channels.

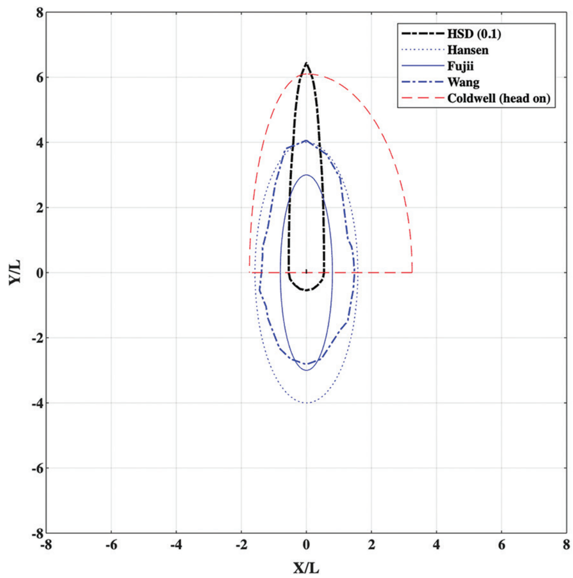


Fig. 10. Comparisons of HSD and restricted area ship domains

One problem is that if many HSDs appear simultaneously, it is easier for OOWs or VTSOs to grasp the dangerous areas quickly. Thus, the method for grouping encounter ships to apply HSD will be presented in the next section.

3.3. IDENTIFYING ENCOUNTER SHIPS BY DBSCAN CLUSTERING

DBSCAN is a clustering algorithm that considers data with a constant density in the same cluster or group [39]. Each data item represented by a coordinate point is divided into three types: core (red dot), boundary (orange dot), and noise (black dot), as shown in Fig. 11.

This algorithm is suitable for ship spatial clustering compared to others because it generates clusters in any shape and can eliminate the noise, which can be considered the singular object in space [40]. There are two main parameters in DBSCAN: Eps, which refers to the radius of the neighborhood area, and MinPts, which refers to the minimum number of objects in a cluster. With clustering, as shown in Fig. 12, the calculation of collision risk can be simplified because it is no longer necessary to consider every ship. In addition, it can also be found that some ships are not in any clusters, are considered noise ships, and are eliminated in the process.

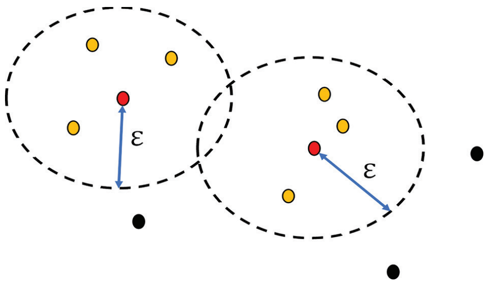


Fig. 11. Illustration of DBSCAN

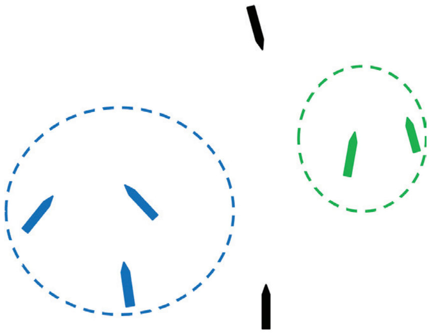


Fig.12. Ship clustering by DBSCAN

Assume x is a data point in the radius Eps of data set X. x can be defined as follows:

$$\begin{aligned} x &= x_{core} \; if \; N(X) \geq MinPts \forall x \epsilon X; \\ x &= x_{border} \; if \; N(X) < MinPts \forall x \epsilon X; \\ x &= x_{noise} \; if \; N(X) \geq MinPts \forall x \epsilon X; \end{aligned} \tag{8}$$

To establish a collision risk prediction map, HSD will be applied to ships identified as core or borders. When the ship leaves its cluster, HSD will not be employed. Although the collision risk around the noise ships is not calculated, they are still tracked until the distance from them to others is smaller than the defined radius. This function will make the high collision risk areas or hot spots easier for users to focus on.

3.1. ESTABLISHING COLLISION RISK PREDICTION MAP

This section proposes a process for presenting the geographical distribution of collision risk or a regional collision risk prediction map. In busy waters, the ship risks colliding with multiple target ships. Therefore, there is a need for a function to estimate the area with a high possibility of collision. The degree of danger increases with the number of target ships at risk of collision. These potential collision areas, or hot spots, are where HSDs overlap, and their indexes (R_{area}) are calculated by Equation 9.

$$R_{area} = \sum_{i=1}^{m} DCR(P_i) \tag{9}$$

where

R_{area} is a collision risk index of overlapped areas;

$DCR(P_i)$ is the collision risk index of point P_i;

m is a number of ships with HSDs that impact point P_i.

This collision risk index of domain-overlapped areas can measure the detail level or degree of collision risk. The greater the index, the more likely the collision will happen if there is no change in the speed and course of at least one ship. If there is no overlap between HSDs, the status of the encounter situation can be suggested as no collision risk. If an overlapped area happens (color becomes hotter), the situation can be considered a pre-collision state if the distance between ships is smaller. The effectiveness of the map with risk prediction function will be verified in Section 4.

4. CASE STUDY AND RESULTS

At present, there are three types of collision risk assessment methods: risk at the micro, macro, and regional level. From a micro point of view, the risk of collision between the pair of ships is usually calculated so that the ships can take action to avoid the collision. However, there are always many ships in the congested sea area, and the possibility of collision between more than two ships will occur frequently. Therefore, this study uses actual AIS data of ships sailing in the waters of Haiphong Ports for experiments to illustrate and validate the proposed model's effectiveness in macroscopic and regional views. Haiphong Ports are located in Northern Vietnam, and there is a vast and rapidly increasing amount of goods due to the development of Vietnam's economy, especially the development of the sea economy. It leads to a rise in the daily flow and density of ships and limits the maneuvering space between ships. Therefore, an urgent need is to analyze and assess the risk of ships colliding in this water.

Geographically, the studied water area is positioned between latitudes 20°46'28.31" N to 20°52'12.57" N, 106°43'36.11" E to 106°55'37.91" E (in Fig. 13).

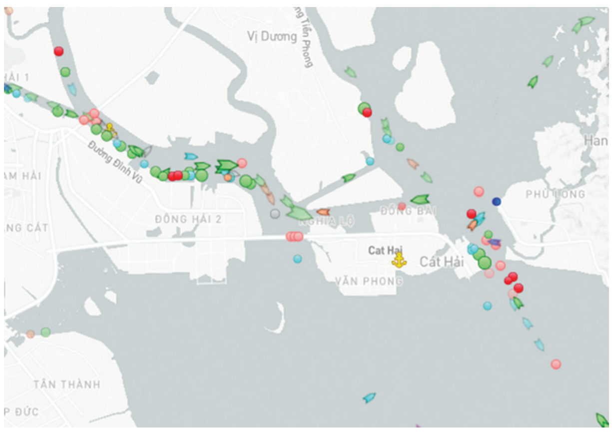


Fig. 13. Study area

In recent years, the growth in the amount of goods and the number of ships passing through Haiphong Ports has caused tremendous pressure on traffic here. Haiphong Ports are along the rivers, with many winding sections, narrow channel width, limited depth, and complicated flow sections intersecting many dangerous areas. There is no TSS here, and the average width of the channel bottom is about 80m. In some areas, avoiding or overtaking each other is difficult, while the number of ships passing to enter the ports increases. There are some passages where ships can only go one way with the control of VTSOs.

The source of the AIS data was on 28th August 2023. To prevent the temporal dispersion of the AIS data, the extracted AIS data is processed through the cleansing and interpolation process described in Section 3.1. Ships' position data in longitude and latitude are converted into Descartes coordinates with axes in nautical miles (NM).

The next step is to group the ships via DBSCAN. The encounter must be formed inside at least two ships, so $MinPts$ should be defined as 2. Eps is the radius at which the ships are connected to form an encounter cluster and varies depending on the situation between the ships, the ship's maneuverability, the sea conditions, meteorological conditions, etc. In general, the designation of Eps should follow the following rules: the Eps value for confined waters should be smaller, and the Eps value for open water should be more significant. For practical purposes in each area, Eps should be matched with the recommendations of experienced captains, OOWs, pilots, and VTSOs of the study area. In this Haiphong Port water case study, the value of Eps was selected as 1.5 NM. After specifying the parameters, DBSCAN is applied to cluster at least two ships into groups and filter out single ships as noises. In previous studies, noise ships were not included, and it is advisable to ignore them, considering the simplification that contributes to the calculations. However, in this study, ships designated as safe are still tracked if they approach other ships and become the border or core of a cluster. The HSD will then be applied to assess the risk. The result is a 2D heat map showing the spatial distribution of the potential risk of collision based on different moments.

First, a numerical simulation that acquires AIS data for three approaching ships was carried out to evaluate the algorithm for determining the macroscopic risk of ship collision based on HSD. The positions and information of the ships at the beginning are shown in Fig. 14 and Table 6. Ship trajectories are illustrated in Fig. 15, with ship A in black, ship B in blue, and ship C in red.

Table 6. Information of ships at the beginning

	Length (m)	Position	Speed (kn)	Course (°)
Ship A	200	20°49.132N 106°53.083E	7,2	081
Ship B	182,9	20°49.658N 106°53.295E	6,0	148
Ship C	222,2	20°48.520N 106°54.341E	6,0	325

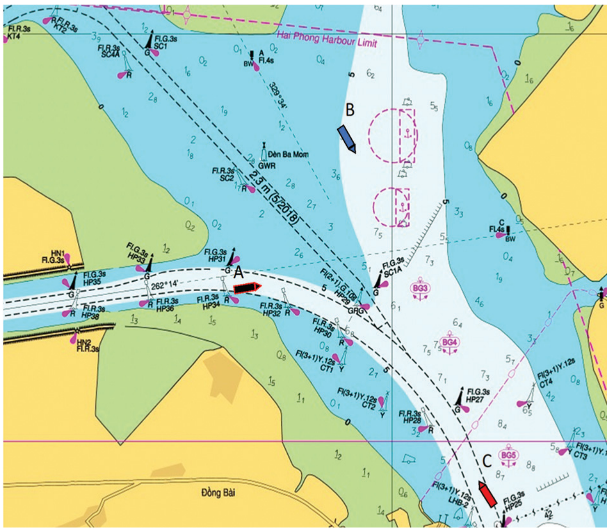


Fig. 14. Positions of ships at the beginning

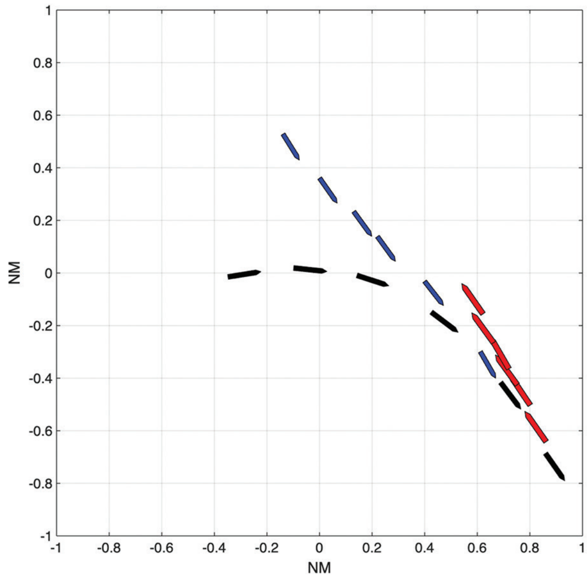


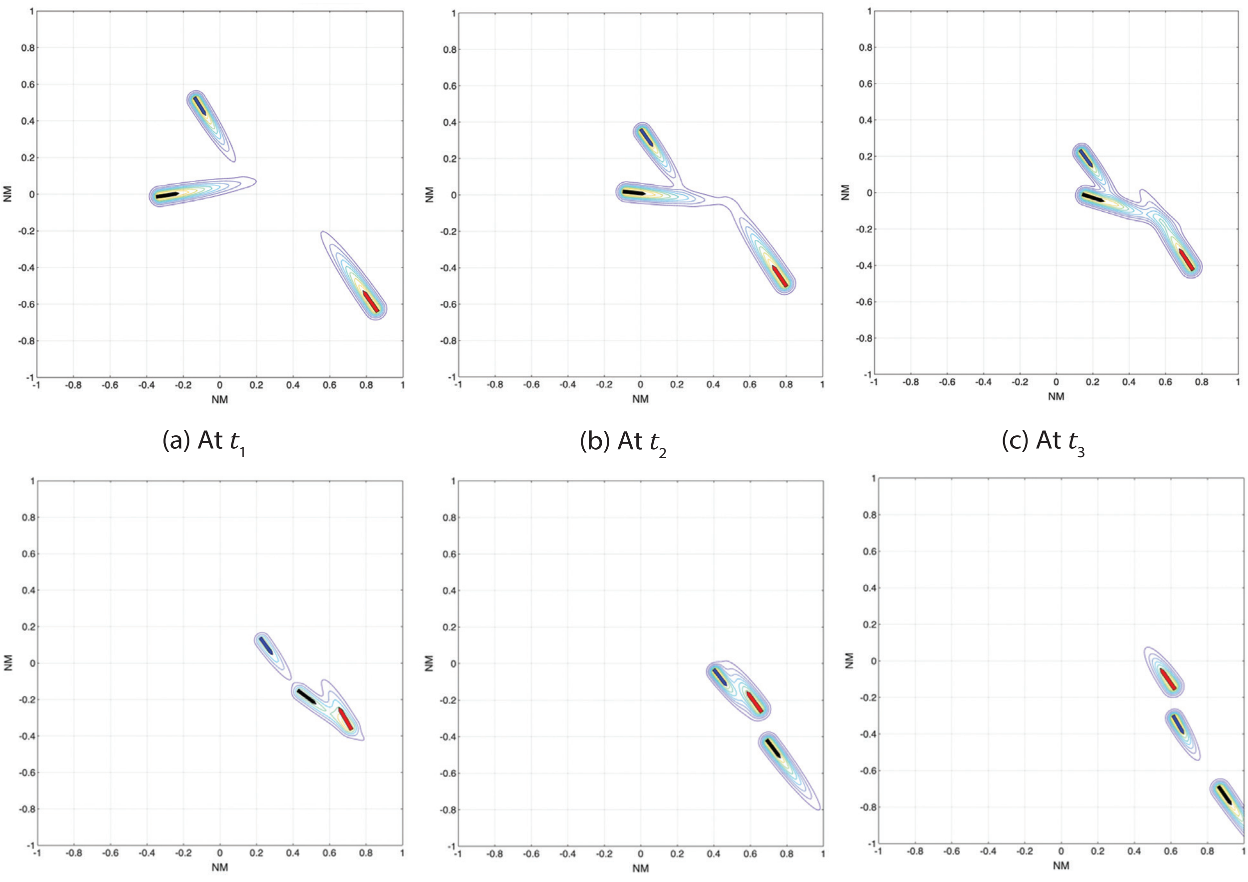
Fig. 15. Trajectories of ships

Due to the distances between ships in the scenario being smaller than 1.5 NM, these ships are clustered in one group, and HSD with $h = 0.2$ is applied to them. At the macroscopic level, the HSDs are presented with iso risk index lines from 0.1 (the most inside line) and 0.9 (the most outside line). The results are shown in Fig. 16 with ship A in black, ship B in blue, and ship C in red. At t_1, when there is no interference between HSDs, the fore parts of the HSDs are about 0.5 NM in length. At t_2, it begins to be observed that there is an overlapped area 1 between the port side of HSD_A and the fore of HSD_B with a value of 0.2 and an overlapped area 2 between the fore of HSD_A and HSD_C with a value of 0.2. Thus, it can be seen that when the overlapped area between HSDs has a value of 0.2, the risk of collision begins to form. At t_3, when the three ships approach each other, the overlapped areas 1 and 2 values increase to 0.5 and 0.4, respectively. The ships began to take action to avoid collision: ship A continued to turn to starboard while ship B and C slowed down. At time t_4, the HSDB became smaller and separated while the HSDA and HSDC still had interference. The most significant value at the overlapped area is 0.6. The two ships, A and C, changed their course to starboard until the HSDs of the two ships separated, and the risk between them did not exist anymore. However, the HSDB and HSDC interfered with each other at t_5, and these two ships continued maneuvering, so there was no interference at t_6.

Although ECDIS can observe areas where ships are assembled, obtaining the exact collision risk values for these locations is impossible. In other words, any location with the same number of ships looks similar, and the collision risk values or indexes of these locations are indistinguishable. Therefore, constructing a Collision Risk Prediction Map is necessary to quickly identify accurate regional collision risk values of high-risk areas by combining multi-ship clusters. Specifically, when the proposed map is adopted, it is possible to obtain an overlapped area of HSDs with value R_{area}. An area with a high rate of R_{area} value during a period can be defined as a hot spot. Each color corresponds to each collision risk level. The areas with more yellow color represent the greater risk.

AIS data from 1005 to 1020 on August 28, 2023, with 12 ships, was processed and then applied DBSCAN and HSD. The results are Collision Risk Prediction Maps of these moments with 5-minute intervals, as shown in Fig. 17). At 1005, 2 clusters of encounter ships were detected. In cluster 1 at the top left corner of the map, two ships were approaching each other. HSDs of two ships overlapped, and this area appeared to be yellow. The index of this area was from 0.88 – 0.93. In cluster 2 on the right side of the map, a ship was passing through the channel, and two other ships were preparing to go in. At this moment, although there was an overlapped area of two HSDs with the value of index 0.33, we can see that this index would increase when the other ship of the cluster came, which would be more dangerous.

At 1010, there was no more overlapped area in cluster 1 due to two ships having taken action to prevent a collision. However, because the distance between these ships is smaller than 1.5 NM (Eps value), HSD was still applied for them. In cluster 2, the index of the overlapped area raised to 0.91 due to the contribution of all three ships. The ships could be advised to start paying attention to it and take substantive actions as soon as possible by judging the index.



(a) At t_1 (b) At t_2 (c) At t_3

(d) At t_1 (e) At t_2 (f) At t_3

Fig. 16. Encounter of ships with HSDs: (**a**) At t_1 there is no interference between HSDs; (**b**) At t_2 there is an overlapped area 1 between the port side of HSDA and the fore of HSDB and an overlapped area 2 between the fore of HSD_A and HSD_C; (**c**) At t_3 three ships continue to approach each other; (**d**) At t_4 there is only interference of HSD_A and HSD_C; (**e**) At t_5 there is only interference of HSD_B and HSD_C; (**f**) At t_6 there is no longer any overlap

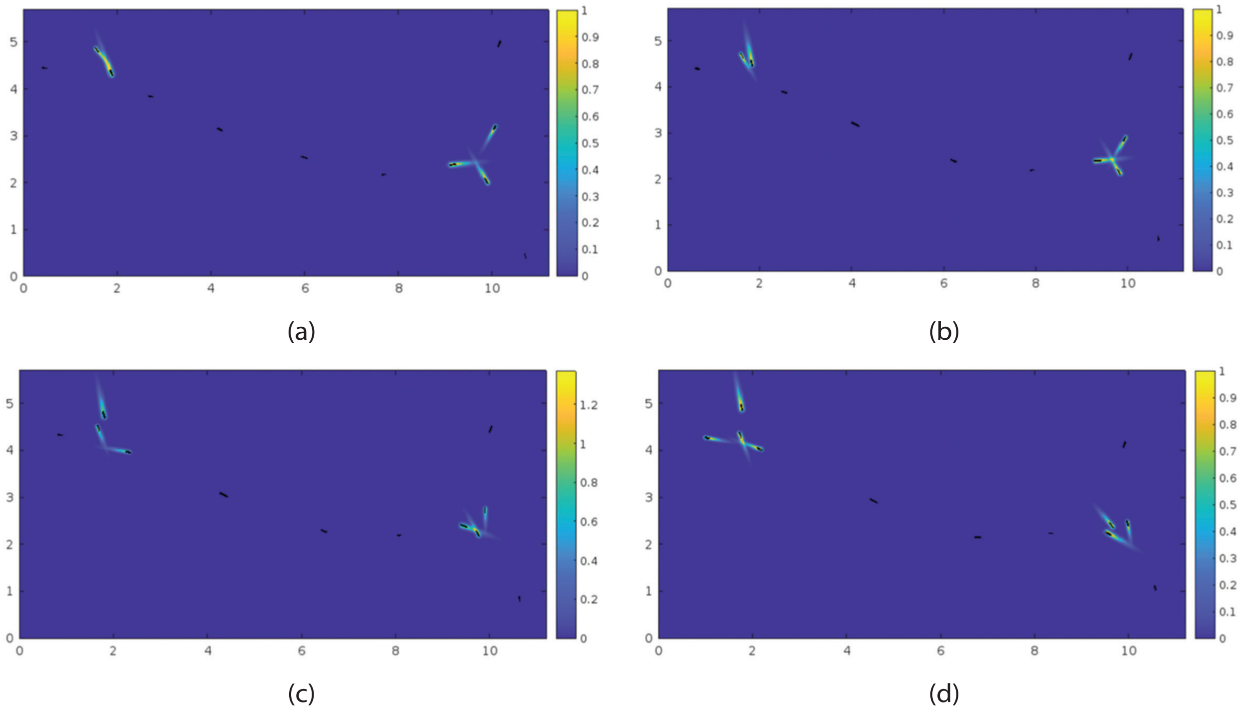


Fig. 17. Collision Risk Prediction Map: (**a**) At 1005, clusters of encounter ships were detected; (**b**) At 1010, there was no overlapped area in cluster 1, and the index of the overlapped area in a cluster; (**c**) At 1015 cluster 1 had one more ship and collision risk became obvious in cluster 2; (**d**) At 1020 four ships formed cluster 1 and HSDs separated in cluster 2

At 1015, cluster 1 had one more ship because a noise ship that had not applied HSD in previous moments started to become a border ship, and its HSD was displayed. Although their HSDs did not violate each other, these ships should be monitored due to their proximity. In cluster 2, before collision risk became apparent, one ship had taken effective collision avoidance measures by changing course and reducing speed. However, the index of the overlapped area still increased to 1.2 due to the close encounter of the other two ships. At this moment, the collision risk was very urgent. At 1020, a ship from the left joined the cluster 1. This cluster now included four ships with the overlapped area of 2 HSDs with an index of 0.85. These two ships were required to take the most helpful action to avoid a collision; otherwise, the collision would occur. In cluster 2, three ships have maneuvered to avoid a collision, and their HSDs are separated, showing safe status. The hot spots area is usually concentrated in the traffic intersection area in the study area.

In narrow areas or narrow channels, especially in the study area (Haiphong waters), it is difficult for ships to cross too close aft side of another. However, this kind of encounter still sometimes happens at the intersections. When these situations happen, the VTSOs should keep continuous communication with all parties, such as captains and maritime pilots, and require all ships to use sound and light signals if necessary. Due to the characteristics of ships and conditions of the channel (depth, width, obstructions, other ships coming…), VTSOs can refer to the proposed framework by observing overlapped areas of HSDs with DCR values (as represented in Table 7) and determine the actions of each ship such as: slowing down or remaining speed, changing or keeping heading… to prevent the collision, and ensure continuous traffic in the monitoring area.

Table 7. DCR range

Status	DCR range	Action
Outlier		Consider as safe but is still tracking
Ship clusterd in group		Pay more attention when HSD is applied
Ship domain overlapped	0 – 0.2	Collision risk begin to exist
Ship domain overlapped	0.2 – 0.5	Collision risk becomes apparent
Ship domain overlapped	0.5 – 1	Collision risk becomes critical
Ship domain overlapped	Greater than 1	Collsion risk becomes extremely critical

According to the COLREGS, the collision risk must be evaluated before determining whether to take a collision-avoidance action or change the current sailing condition. Before the risk of collision exists, ships are free to take any action, when a ship is located in a safe area and is not close to any ship. When a ship is clustered with others, they are advised to pay attention. In this situation, the HSD is applied to help visualize VTSOs and OOWs. When HSDs are overlapped, the overlapped area has $0 < DCR \leq 0.2$, the risk of collision first begins, and the ships must take early and substantial action to achieve a safe passing distance.

When $0.2 < DCR \leq 0.5$, ships should take more appropriate actions in compliance with the COLREGs to avoid collision. Similarly, when $DCR > 0.5$, ships must take the best aid to avoid a collision; otherwise, there will be a collision.

In particular, the proposed framework focuses on the index of overlapped areas of the ship domains to predict the location of collision, which is different from a previous method, which considered the position of the target ship inside the domain of its ship to evaluate collision risk. In fact, in congested areas, it is necessary to construct a framework that can detect risk as early as possible. The situation when a ship violates another ship's domain is urgent, and ships may not have enough time to act. This Collision Risk Prediction Map is accurate and stable by transforming domain overlap problems into the distribution of high-risk areas. The final results of the case study demonstrate that the parameters are appropriate; however, other sea waters need to adopt another value of area parameter due to their specific characteristics and conditions as well as the requirements and policies of the maritime traffic surveillance systems. The proposed framework shows its collision risk detection function at some past moments. If real-time AIS data is applied, the whole procedure of data processing and clustering encounter ships by DBSCAN and HSD should be implemented continuously due to the change of ships' positions, speeds, headings, and traffic density. As a result, the collision risk prediction map can change constantly. Finally, the detection of high-risk areas is a dynamic process over time.

5. CONCLUSIONS

This study proposes the Collision Risk Prediction Map based on AIS data. To simplify the computation and make it quicker to detect the hot spots, DBSCAN was used to cluster the ships in the water area, and the contribution of each ship to potential collision within the cluster is calculated as a function of the Heat Ship Domain. This domain is dynamic and expresses the risk around the ship by index. Collision risk was identified if an overlapped area between HSD happens. Finally, the geographical distribution of collision risk was visualized to establish a collision risk prediction map. To validate the effectiveness of the new map, a case study was conducted in the waters of Haiphong Ports in Vietnam. The results show that the visualization obtained by the framework can effectively reflect the collision risk of specific waters through the index in three perspectives: micro, macro, and region. Unlike other collision risk identification models, the parameter can be easily adjusted based on experts' knowledge of the study area. The proposed framework can help maritime traffic operators or administrations better understand the overall collision risk and its distribution in the water during collision risk monitoring.

6. ACKNOWLEDGEMENT

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