Antenna Array Diagnosis in the Presence of Unknown Mutual Coupling using Optimization Technique

Original Scientific Paper

Oluwole John Famoriji*
Department of Electrical and Electronic Engineering Technology,
University of Johannesburg, P.O. Box 524, Auckland Park,
Johannesburg 2006, South Africa.
famoriji@mail.ustc.edu.cn

Thokozani Shongwe
Department of Electrical and Electronic Engineering Technology,
University of Johannesburg, P.O. Box 524, Auckland Park,
Johannesburg 2006, South Africa.
tshongwe@uj.ac.za

*Corresponding author

Abstract *– Antenna array diagnosis is an important operation in communication systems, whenever element (s) failure in the array that worsening the projected radiation pattern occur. There are various diagnostic techniques found in literature that employ compressive sensing. Conversely, the techniques are based on easy formulation of array factor with no incorporation of mutual coupling existing between the radiators. This article shows how this deficiency lead to defective and bad diagnosis when there is presence of mutual coupling using port-level coupling matrix and average embedded antenna pattern. Furthermore, the element excitations are optimized to reduce the effect of mutual coherence of system measurement matrix, causing reduced measurements required for effective fault detection. Numerical simulation and experimental results demonstrate how the incorporation of mutual coupling generates an adequate and reliable array diagnosis, which are not found in literature. For instance, when fault number is set at 5, and SNR equals 10 dB, the smallest measurements needed for the diagnosis, which is the most effective diagnosis, are achieved when the optimized excitations are used. In conclusion, the implementation of the developed framework using measurement probe in space, shows enough results towards the practical deployment for antenna systems in wireless communication system.*

Keywords*:* *Antenna arrays, array signal processing, array diagnosis, optimization techniques, SNR, mutual coupling*

Received: Received: February 15, 2024; Received in revised form: March 30, 2024; Accepted: April 19, 2024

1. INTRODUCTION

Antenna array diagnosis is an important research topic that finds application in civilian and military. Present and upcoming technology use larger number of elements in the active arrays. For instance, large number of elements is employed in massive MIMO (multiple input multiple output), full MIMO systems, and telecommunication devices that employ sophisticated arrays. Hence, the demand for a reliable antenna array diagnosis is an inevitable task to rectify the distorted radiation characteristics because of element (*s*) failure [1-6]. In addition, fault diagnosis is important in 5G wireless systems, where a very large number of elements are needed to satisfy the required reconfigurability and high radiation behaviour [2]. However, the more the number of elements in beamforming configuration, the more the probability of failed element (s). Therefore, an effective and highly reliable fault diagnosis method remains important, because replacement operation and manual dismantling take a lot of time, costly, and not feasible in satellite communications [2, 4]. Compressive sensing (*CS*) technique has been adapted to fault diagnosis in antenna arrays, because the number of failed radiators is assumed and always smaller than the total number of radiators in the antenna array.

Some array diagnosis algorithms, such as the backward transformation method (BTM) [5] and matrix method (MM) [6], efficiently identify the locations and excitations of faulty elements using discrete Fourier transform (DFT) and matrix inversion techniques, respectively. However, these approaches are highly susceptible to noise and require a minimum number of sampling points. Specifically, for MM, the sampling points should equal or exceed the number of elements in the array to prevent ill-conditioned matrices during solving. Additionally, diagnostic methods employing intelligent optimization algorithms, such as genetic algorithms [7] and artificial neural networks [8], are computationally intensive.

Hence, the primary challenge in array diagnosis currently revolves around selecting an appropriate method to swiftly identify the faulty elements within the array. Additionally, this solution must exhibit low sensitivity to noise and provide flexibility in the number of sampling points utilized. The matrix pencil method (MPM) was originally introduced for estimating the parameters of complex exponential and attenuation exponential signals [8, 9]. However, a drawback of MPM is its limitation in handling the continuous distribution of synthesised element locations [7-10], rendering it unsuitable for array synthesis featuring elements positioned at fixed grid coordinates.

Furthermore, there are different array diagnostic methods that employed CS [3], [8-18], even with validation with experiments [18]. The array diagnosis is demonstrated in most approaches in literature employing recovery sparse solution from small number of measurements to show the situation (healthy of faulty) of elements. Some methods advised array diagnosis employing measured data taken from a point with different excitations [18], [19]. Conversely, each technique uses easy array factor dependent far-field model. Simplicity is offered, but non-ideal negligibility is contentious towards array diagnosis in practical sense, specifically when inter-element spacing is in smaller. Some recent literature modelled multipath channel to the fixed probe when there are faults [20], [21], but there is no work, to the best of authors' knowledge, that considers the mutual coupling (MC) impacts [22] in fault identification with the employment of a fixed receiver probe for measurements.

This article demonstrates why MC should be considered in fault diagnosis, and if not accounted for causes poor diagnosis. It is demonstrated how the proposed fault diagnosis method that employs a fixed probe and excitations optimization achieves optimal performance simply even while MC is considered. Furthermore, numerical experiment is demonstrated and the results involved AEAP (average embedded antenna pattern) and PLCM (port-level coupling matrix) methods; implying the proposed method provides effective and reliable array diagnosis. In addition, two MC modeling methods are presented in this article, they can be applied based on array patterns and available data about the antenna array to the user.

2. SYSTEM MODEL

This section provides the analysis of the faulty array at far-field, and the proposed fault detection approach.

2.1. FAULTY ANTENNA ARRAY AT FAR FIELD

The Based on the linearity feature of Maxwell's equations; that EM field originated from antenna array is formulated as a linear excitations juxtaposition of elements in the array, $E(r)=\sum_{j=1}^{N} \alpha_j (r) y_j$, here, $E(r)$ denotes the EM wave at point r, while y_j are the excitations, $\alpha_j (r)$ are the resulted combination of the coefficients that consist the information regarding the EM surrounding of the array element and the measurement setup, N is the number of elements in the array. Since the aim of this article is to conduct fault diagnosis with inter-element MC, then the computational steps that provide the system model can be outlined as follows.

2.1.1. How is the Element' Fault Modeled?

On this issue, the excitation y_j is replaced by $y_j \delta_j$, $\delta_j \in C$ show the state of fault of the element. For instance, $\delta_j=1$ means there is no fault, while $\delta_j=0$ means a deceased element.

2.1.2. MC Modeling

In simple term, if coupling is neglected (from element pattern isolation technique), α_j is made up of parameters that determine the gain of the element, and phase due to distance between the measurement point and location of the element. Conversely, if the α_j parameters are calculated using active element patterns [23], then the effects of MC are incorporated fully compared to the N full wave EM simulations of all measurement. Many more techniques are found in literature, which is involve AEAP [24] and PLCM methods [25-29]. Then, the measured field is expressed as [12]

$$E(r) = \sum_{j=1}^{N} y_j \alpha_j(r) \delta_j, \tag{1}$$

while the parameter expression of α_j is a function of the particular model employed for MC impacts.

2.2. PROPOSED FAULT DETECTION TECHNIQUE

By critical examination of Eq. (1), it can be observed that for a fixed measurement location, the α_j parameters are unchanged, hence the 'r' argument is dropped. Consequently, a M measurements vector is built, $\tilde{x}$, and by relation to the excitation, we have $\tilde{x}=\sum_{i=1}^{N} y_i^{(i)} \alpha_i \delta_i$, where $y_i^{(i)}$ represents element i excitation for measurement j, which formulate the following model [12]

$$\tilde{x} = \begin{bmatrix} y^{(1)} \\ \vdots \\ y^{(M)} \end{bmatrix} \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \alpha_N \end{bmatrix} = \underbrace{Y diag(\alpha)}_{D} \delta, \tag{2}$$

where $y^{(j)} \in C^{1 \times N}$ has the excitations of the element for the j^{th} measurement, leading to total excitation matrix, $Y \in C^{M \times N}$, and $\delta \in C^{N \times 1}$ denotes the vector of the state of fault.

Generally, in fault diagnosis, it is usually assumed that reference measurements, such as $x^{(R)}$, are accessible for a particular state of elements, $\delta^{(R)}$, which corresponds to array without fault. Another assumption here is the sparsity of the number of failed radiators as to the reference, so, following the formulation of vector from the differential measurement, $x = x^{(R)} - \tilde{x}$, the resulting problem requiring solution for the diagnosis of faults becomes [12]

$$\min_p \|p\|_0 \,, s.t. \|x - Dp\| < \mu, p = \delta^{(R)} - \delta, \qquad (3)$$

μ is a term that shares proportionality to standard deviation of the measurement noise.

The sparse recovery requires that D (sensing matrix) exhibits low mutual coherence [27, 28]. For mutual coherence, we optimized the excitation matrix Y via the approximation of a Grassmannian matrix by alternating method [27], which exhibits performance improvement in comparison to randomized Y [29]. D is a diagonal matrix that multiplies Y, the mutual coherence of matrix D equals that of the Y, hence the optimization of the mutual coherence heedless to the kind of MC model employed. The main knowledge acquired is the constancy of the linear coupled forward model for various excitations. Note, this is impossible for multiple points measurements. Using the standard techniques to transform the unconstrained nonconvex optimization problem [15, 16, 30] we have [12]

$$\min_p \|x - Dp\|_2^2 + \gamma \, \|p\|_z^z, 0 < p < 1, \qquad (4)$$

γ is the hyperparameter (empirical). The problem is resolved via the iterative reweighted l_1 minimization [30-37] implemented using alternating direction method of multipliers (ADMM) [38].

3. NUMERICAL SIMULATIONS, RESULTS, AND DISCUSSION

This section demonstrate and verifies the effectiveness of the proposed diagnosis method. 100 elements constituted of WR90 open-ended waveguide array working at 27 GHz as depicted in Fig. 1 [5]. The radiation characteristics at no fault at $\phi = 0^0$ is given in Fig. 2. The array aperture size is 24.88 x 12.18 mm^2 and the spacing between elements in both x and y axis is λ and 0.5λ, respectively. The Ansys HFSS v.19 software was used for the simulations.

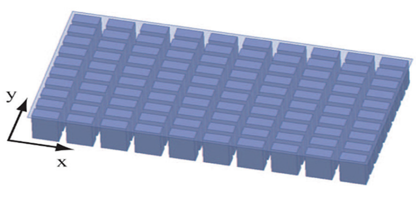


Fig. 1. Open ended waveguide array used for diagnostic demonstration [5].

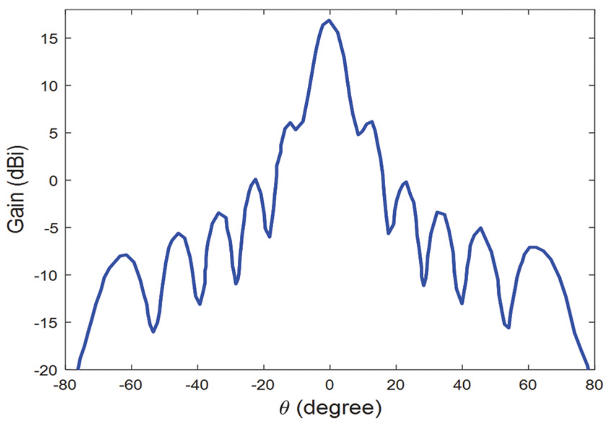


Fig. 2. Simulated radiation pattern of antenna array without element failure at principal plane $\phi = 0^0$[5].

The waveguide is excited using a 50 Ω generator impedance and designed on the substrate ASTRA MT77 with relative permittivity $\varepsilon_r = 3$ and loss factor tan $\delta = 0.0017$ to have an impedance of 50 Ω. Also, the physical overlapping of elements at smaller value spacing between elements, such as $d = 0.45\lambda$, and broadband radiation pattern used in wireless networks is ensured.

Next is to present the results of the diagnosis using the complex E-field (E_y) measured data obtained at fixed point via excitations optimization. The measurements of field of the faulty and healthy arrays (i.e. the forward models of α_j in Eqn. (1)) of isolated pattern method (using the array factor method [33]), average embedded pattern method (using Eqn. (11) of [23]), and coupling matrix method (using Eqn. (A9) of [25]) were simulated using Matlab Antenna Toolbox. It is important to state that the MC model employed in this article is not restricted, any MC model can be employed by appropriate computation of α_j based on the proposed scheme (as in Eqn.) (2).

The point of measurement is fixed at a spherical angular point $(\theta_0, \phi_0) = (0,0)$, $r = 1000\lambda$ (in z-direction). Using these measurements, the solution of the fault is gotten by the iterative reweighted $\ell 1$ minimization. The hyperparameters γ of Eqn. (4) is given as $\gamma = 0.25\|D^H x\|_\infty$ and $z = 0.5$ (z quasi-norm) for each fault diagnosis result. The γ is an empirical value obtained from a grid search from $0.1\|D^H x\|\infty$ to $\|D^H x\|_\infty$. The upper limit $\|D^H x\|_\infty$ is explained in [27]. The phase and amplitude of excitations are set at [0, 2π], [0, 1], respectively. Both randomized and optimized excitations are quantified into six-bit phase and amplitude, while we obtain randomized excitations via multinomial probability distribution.

The solution that is recovered is designed into binary numbers using the threshold of the actual part by 1/2, i.e. for nth antenna $\delta_n = 0$. Fault recovery is successful when the accurate faulty elements reconstruction and corresponding position is attained. It is important to state that thresholding action is unnecessary when dealing with non-binary faults. Rate of successful recovery (RSR) metrics is employed to present the results, and the realizations percentage leading to successful recovery. The results are computed using 600 Monte-Carlo simulations with random fault positions. The findings originated from the proposed method are outlined below.

3.1. ENHANCEMENT IN FAULT DETECTION USING EXCITATIONS OPTIMIZATION

The proposed method incorporates the impact of MC. This implies there should be similar improvement in performance when excitations optimization with respect to random excitations is used. This is validated via performance analysis of fault diagnosis with coupling matrix model, without and with excitation matrix Y optimization at 10 dB signal-to-noise ratio (SNR). The excitation matrix Y optimization is confirmed to be useful for coupling matrix method as depicted in Fig. 3. When the average pattern method is used as the MC model, similar results were attained. This is an important result as all the merits of the optimized element excitation method continuously apply when the impacts of MC are considered. Similar findings were observed at higher SNRs, and there is an improvement generally in the RSR for a particular number of faults.

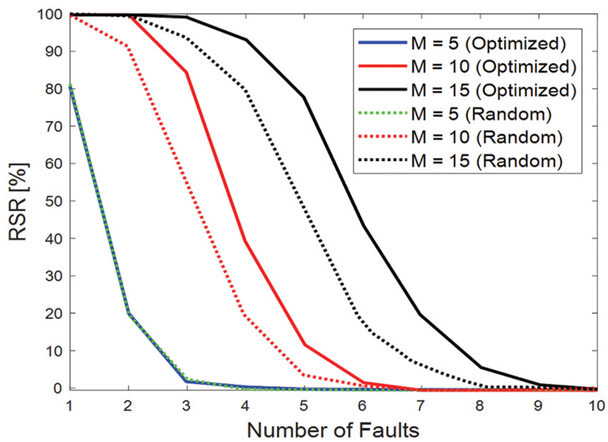
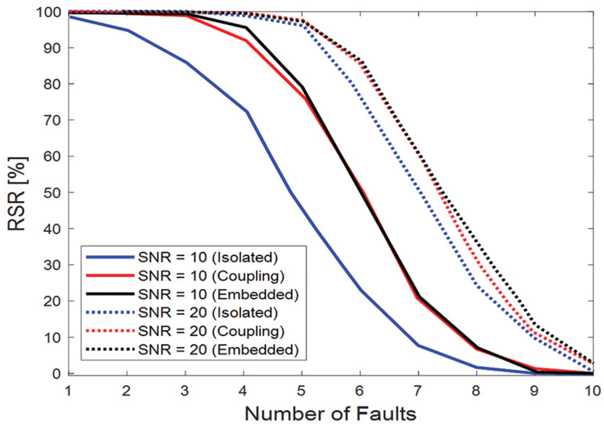


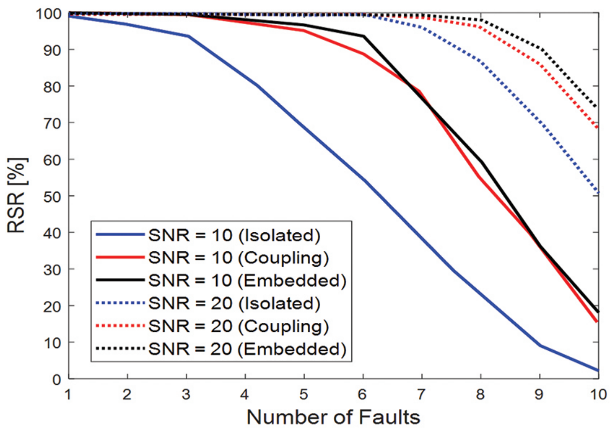
Fig. 3. Plot of RSR for coupling matrix method using iterative reweighted $\ell 1$ minimization, at N=100, d=0.45λ, and SNR=10 dB

3.2. ENHANCEMENT IN FAULT DETECTION WITH MUTUAL COUPLING

The RSR percentage versus the faults number via various coupling models with iterative reweighted $\ell 1$ minimization and excitations optimization for 10 dB and 20 dB, respectively, is depicted in Fig. 4. Fig. 4 are for two M, and it can be observed that evident reliability improvement in fault diagnosis is observed when MC impacts are considered in both measurement cases. For instance, when SNR is 10 dB with 15 measurements, M, and 3 faults, RSR of 100 is attained in the presence of MC, as against the 85 RSR without MC. In addition, when SNR is 10 dB with 20 measurements, M, and 4 faults, 98 RSR is attained in the presence of MC as against the 83 RSR without MC.



(a)



(b)

Fig. 4. Plot of RSR using iterative reweighted $\ell 1$ minimization, at N=100, d=0.45λ, and excitations optimization for 2 measurements, M, (a) M=15 (b) M=20.

Furthermore, the RSR accuracy difference when the MC is considered is higher than when low SNR measurements are used. For instance, comparing the performance of high and low SNRs in Fig. 4, the high SNR line shoot out more than that of low SNRs. For high SNR, an improved RSR is observed when the presence of MC is considered, even at bigger number of faults. For instance, when SNR is 20 dB, and measurements, M is 20 including 10 faults; isolated pattern method attains 50 percent RSR, while coupling models attain about 71% RSR.

3.3. VARYING THE DISTANCE BETWEEN ELEMENTS: ANALYSIS

When the antenna elements are closer, the impact of MC is more [34], it is expected to have all the methods converge to close performance at higher spacing between elements. As this can be demonstrated in the results, it is valid based on least measurements M needed correct diagnosis of faults. Tables I to IV depict the measurements number, M required to achieve 90% RSR and 98% RSR when array diagnosis is conducted for various spacings between elements with the developed models and random/optimized excitations. Fault number is set at 5, and SNR equals 10 dB. The smallest measurements M needed for the diagnosis, which is the most effective diagnosis, are achieved when the optimized excitations are used (as per Table 1 & 3).

Generally, the AEAP models and coupling matrix with excitations optimization exhibit better options of finding fault in this scenario. Conversely, it becomes important to state that the coupling matrix method is of the assumption that the array impedance matrix is readily available for the user. When it is not available, the average embedded pattern model is recommended for practical application with large array aperture, because it requires only the response of the embedded pattern of the element at the center, easy to achieve.

Table 1. The required number of measurements to achieve 98% RSR with varying spacing d and 5 number of faults for optimized excitation

Spacing, d	**0.45λ**	**0.95λ**	**1.45λ**	**1.95λ**
Embedded Pattern	25	19	19	19
Coupling Matrix	23	20	21	19
Isolated	0	32	40	19

Table 2. The required number of measurements to achieve 98% RSR with varying spacing d and 5 number of faults for random excitation

Spacing, d	**0.45λ**	**0.95λ**	**1.45λ**	**1.95λ**
Embedded Pattern	28	25	24	25
Coupling Matrix	31	24	27	23
Isolated	0	35	48	25

Table 3. The required number of measurements to achieve 90% RSR with varying spacing d and 5 number of faults for optimized excitation

Spacing, d	**0.45λ**	**0.95λ**	**1.45λ**	**1.95λ**
Embedded Pattern	18	17	17	17
Coupling Matrix	18	17	17	17
Isolated	38	22	27	17

Table 4. The required number of measurements to achieve 90% RSR with varying spacing d and 5 number of faults for random excitation

Spacing, d	**0.45λ**	**0.95λ**	**1.45λ**	**1.95λ**
Embedded Pattern	22	20	20	19
Coupling Matrix	24	20	22	21
Isolated	45	25	31	20

4. CONCLUSION

In this paper, two methods of adding the impact of MC for effective and more reliable antenna array fault diagnosis via a notable CS method that employs fixed probe based measurements, and excitations optimization. Based on the accessible data about the array, and corresponding characteristics, both methods can be employed. For a fairly large array, AEAP method is an appropriate forward model, and coupling matrix method is appropriate for a particular antenna group. The superiority of the forward models, which incorporate the impact of MC, have been demonstrated and the shortcoming of a forward model without the MC influences incorporation has been presented. For instance, when fault number is set at 5, and SNR equals 10 dB, the smallest measurements needed for the diagnosis, which is the most effective diagnosis, are achieved when the optimized excitations are used. The proposed technique is verified and demonstrated to be highly correct, and reliable in fault finding in antenna arrays where the impacts of MC are cannot be ignored. Finally, the proposed technique is more practical and recommended for identification of faults in antenna arrays.

5. REFERENCES

[1] O. M. Bucci, M. D. Migliore, G. Panariello, P. Sgambato, "Accurate diagnosis of conformal arrays from near-field data using the matrix method", IEEE Transactions on Antennas and Propagation, Vol. 53, No. 3, 2005, pp. 1114-1120.

[2] A. Massa, P. Rocca, G. Oliveri, "Compressive sensing in electromagnetics—A review." IEEE Antennas and Propagation Magazine, Vol. 57, 2015, pp. 224-238.

[3] M. D. Migliore, "A compressed sensing approach for array diagnosis from a small set of near-field measurements", IEEE Transactions on Antennas and Propagation, Vol. 59, 2011, pp. 2127-2133.

[4] A. Massa, M. Bertolli, G. Gottardi, A. Hannan, D. Marcantonio, G. Oliver, A. Polo, F. Robol, P. Rocca, F. Viani, "Compressive sensing as applied to antenna arrays: Synthesis, diagnosis, and processing", Proceedings of the IEEE International Symposium on Circuits and Systems, Florence, Italy, 27-30 May 2018, pp. 27-30.

[5] J. J. Lee, E. M. Ferren, D. P. Woollen, K. M. Lee, "Near-field probe used as a diagnostic tool to locate defective elements in an array antenna", IEEE Transactions on Antennas and Propagation, Vol. 36, No. 6, 1988, pp. 884-889.

[6] O. M. Bucci, M. D. Migliore, G. Panariello, P. Sgambato, "Accurate diagnosis of conformal arrays from near-field data using the matrix method", IEEE Transactions on Antennas and Propagation, Vol. 53, No. 3, 2005, pp. 1114-1120.

[7] R. Iglesias, F. Ares, M. Fernandez-Delgado, J. A. Rodriguez, J. Bregains, S. Barro, "Element failure de-

tection in linear antenna arrays using case-based reasoning", IEEE Antennas and Propagation Magazine, Vol. 50, No. 4, 2008, pp. 198-204.

[8] X. Wang, K. Konno, Q. Chen, "Diagnosis of array antennas based on phaseless near-field data using artificial neural network", IEEE Antennas and Propagation Magazine, Vol. 69, No. 7, 2021, pp. 3840-3848.

[9] O. J. Famoriji, T. Shongwe, "An effective antenna array diagnosis method via multivalued neural network inverse modeling approach", Advanced Electromagnetics, Vol. 10, No. 3, 2021, pp. 58-70.

[10] O. J. Famoriji, T. Shongwe, "A Recovery Performance Study of Compressive Sensing Methods on Antenna Array Diagnosis from Near-Field Measurement Data", Applied Computational Electromagnetics Journal, Vol. 36, No. 8, 2021, pp. 1-7.

[11] A. F. Morabito, A. R. Lagana, T. Isernia, "Optimizing power transmission in given target areas in the presence of protection requirements", IEEE Antennas and Wireless Propagation Letters, Vol. 14, 2014, pp. 44-47.

[12] A. Fadamiro, A. Semomhe, O. J. Famoriji, F. Lin, "A multiple element calibration algorithm for active phased array antenna", IEEE Journal of Multiscale and Multiphysics, Vol. 4, 2019, pp. 163-170.

[13] M. D. Migliore, "Array diagnosis from far-field data using the theory of random partial Fourier matrices", IEEE Antennas and Wireless Propagation Letters, Vol. 12, 2013, pp. 745-748.

[14] T. Ince, G. Ogucu, "Array failure diagnosis using nonconvex compressed sensing", IEEE Antennas and Wireless Propagation Letters, Vol. 15, 2015, pp. 992-995.

[15] O. Famoriji, Z. Zhang, A. Fadamiro, Md. Ali, R. Zakariyya, F. Lin, "Planar array diagnostic tool for millimeter wave communication systems", Electronics, Vol. 7, No. 12, 2018, pp. 1-23.

[16] B. Fuchs, L. Le Coq, M. D. Migliore, "Fast antenna array diagnosis from a small number of far-field measurements", IEEE Transactions on Antennas and Propagation, Vol. 64, No. 6, 2016, pp. 2227-2235.

[17] O. J. Famoriji, A. Fadamiro, Z. Khan, Z. Zhang, F. Lin, "Active antenna array diagnosis from far-field measurements", Proceedings of the International Conference on Integrated Circuits, Technologies and Applications, Beijing, China, 21-23 November 2018, pp. 1-4.

[18] C. Xiong, G. Xiao, Y. Hou, M. Hameed, "A compressed sensing based element failure diagnosis method for phased array antenna during beam steering", IEEE Antennas and Wireless Propagation Letters, Vol. 18, No. 9, 2019, pp. 1756-1760.

[19] W. Li, W. Deng, Q. Yang, X. Zhang, J. Zhang, M. D. Migliore, "A Hybrid Method for Array Diagnosis Using Random Perturbation-Convex Local Minimizer", Proceedings of the IEEE Radar Conference, Boston, MA, USA, 22-26 April 2019, pp. 1-5.

[20] S. Costanzo, A. Borgia, G. Di Massa, D. Pinchera, M. D. Migliore, "Radar array diagnosis from undersampled data using a compressed sensing/sparse recovery technique", Journal of Electrical and Computer Engineering, Vol. 2013, 2013, pp. 324-331.

[21] R. Sun, W. Wang, L. Chen, G. Wei, W. Zhang, "Blind diagnosis for millimeter-wave large-scale antenna systems", IEEE Communications Letters, Vol. 25, No. 7, 2021, pp. 2390-2394.

[22] G. Medina, A. S. Jida, S. Pulipali, R. Talwar, N. A. J, T. Y. Al-Naffouri, A. Madanayake, M. E. Eltayeb, "Millimeter-Wave Antenna Array Diagnosis with Partial Channel State Information", Proceedings of the ICC IEEE International Conference on Communications, Montreal, QC, Canada, 14-23 June 2021, pp. 1-5.

[23] I. Gupta, A. Ksienski, "Effect of mutual coupling on the performance of adaptive arrays", IEEE Transactions on Antennas and Propagation, Vol. 31, No. 5, 1983, pp. 785-791.

[24] Y. Zhang, H. Zhao, "Failure diagnosis of a uniform linear array in the presence of mutual coupling", IEEE Antennas and Wireless Propagation Letters, Vol. 14, 2015, pp. 1010-1013.

[25] A. Ludwig, "Mutual coupling, gain and directivity of an array of two identical antennas", IEEE Transactions on Antennas and Propagation, Vol. 24, No. 6, 1976, pp. 837-841.

[26] D. Kelley, W. Stutzman, "Array antenna pattern modeling methods that include mutual coupling

effects", IEEE Transactions on Antennas and Propagation, Vol. 41, No. 12, 1993, pp. 1625-1632.

[27] T. Su, H. Ling, "On modeling mutual coupling in antenna arrays using the coupling matrix", Microwave and Optical Technology Letters, Vol. 28, 2001, pp. 231-237.

[28] B. Clerckx, C. Craeye, D. Vanhoenacker-Janvier, C. Oestges, "Impact of Antenna Coupling on 2 x 2 MIMO Communications", IEEE Transactions on Vehicular Technology, Vol. 56, No. 3, 2007, pp. 1009-1018.

[29] C. Craeye, D. Gonzalez-Ovejero, "A review on array mutual coupling analysis", Radio Science - RADIO SCI, Vol. 46, 2011, pp. 23-33.

[30] M. Elad, "Sparse and Redundant Representations: From Theory to Applications in Signal and Image Processing", 1_{st} Edition, Springer, 2010.

[31] E. Candes, J. Romberg, "Sparsity and incoherence in compressive sampling", Inverse Problems, Vol. 23, No. 3, 2007, p. 969.

[32] E. J. Candes, T. Tao, "Near-optimal signal recovery from random projections: Universal encoding strategies?", IEEE Transactions on Information Theory, Vol. 52, No. 12, 2006, pp. 5406-5425.

[33] Q. Lyu, Z. Lin, Y. She, C. Zhang, "A comparison of typical p minimization algorithms", Neurocomputing, Vol. 119, 2013, pp. 413-424.

[34] S. Foucart, M.-J. Lai, "Sparsest solutions of underdetermined linear systems via q-minimization for lq-minimization for $q \leqslant 1$", Applied and Computational Harmonic Analysis, Vol. 26, 2009, No. 3, pp. 395-407.

[35] S. Boyd et al. "Distributed optimization and statistical learning via the alternating direction method of multipliers", Foundations and Trends in Machine learning, Vol. 3, No. 1, 2011, pp. 1-122.

[36] R. C. Hansen, "Phased Array Antennas", John Wiley & Sons, 2009.

[37] O. J. Famoriji, T. Shongwe, "Electromagnetic machine learning for estimation and mitigation of mutual coupling in strongly coupled arrays", ICT Express, Vol. 9, No. 1, 2023, pp. 8-15.

[38] O. J. Famoriji, T. Shongwe, "Antenna array diagnosis via smart sensing of electromagnetics with learnable data acquisition and processing", Proceedings of the AEIT International Annual Conference, Rome, Italy, 3-5 October 2023.