Energy Efficiency of the Passive Hybrid Battery Supercapacitor System: An Analytical Approach

Original Scientific Paper

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Abstract *– Under dynamic load, passive hybrid battery supercapacitor systems (HBSSs) have numerous advantages compared to stand-alone battery systems (SABS). The highlighted benefits include a lower voltage drop at the HBSS terminals, lower current and thermal stress on the battery, and higher energy efficiency. A novel set of analytical expressions is introduced for the analysis of energy efficiency of the passive HBSS in relation to the SABS. Using these expressions, energy efficiency of the passive HBSS was analyzed for different dynamic load parameters. Compared to existing approaches, the proposed approach that uses the introduced analytical expressions provides a better insight into the impact of battery and supercapacitor parameters on the energy efficiency of the passive HBSS in relation to the SABS. Additionally, the proposed approach is suitable for quick determination of HBSS behavior and energy efficiency under dynamic load. This contributes to the advancement of the sizing of HBSS components and optimization practices, ultimately leading to improved energy use. The analysis showed that in cases where the dynamic component of the load current is a very small part of the total load current, or slowly varying load currents, the passive HBSS in relation to the SABS has no advantages in terms of energy efficiency. The analysis showed that the HBSS is more suitable than the SABS in applications where the dynamic component of the load current is significant. The findings also indicate that there is an interval of the battery and supercapacitor parameters which is optimal for the sizing of the passive HBSS from a techno-economic aspect.*

Keywords: *analytical, battery, dynamic, energy efficiency, hybrid battery supercapacitor system, supercapacitor*

Received: June 4, 2024; Received in revised form: August 14, 2024; Accepted: August 19, 2024

1. INTRODUCTION

From the perspective of energy efficiency, energy sources such as photovoltaics (PV), batteries, wind turbines, diesel generators, supercapacitors, etc. have significantly different energy efficiency for dynamic and static loads. In general, system losses are higher with dynamic loads compared to static loads. Additionally, when supplying dynamic loads, energy sources experience increased current and thermal stresses and higher voltage drops. By hybridizing different energy sources, the desired characteristics of individual energy sources can be exploited in certain applications [1-3]. By applying optimization techniques, adequate power management, the optimal configuration of such systems, dynamics and energy flows can be achieved [3, 4]. It is also possible to reduce current and thermal stresses of components within hybrid systems as well as voltage drops at the terminals of the hybrid system. One such hybrid system, the so-called passive hybrid battery and supercapacitor system, is described in detail in this paper.

As energy storage devices, batteries [5, 6] and supercapacitors (SCs) [7-12] are particularly suitable for static and dynamic loads, respectively. Despite the fact that supercapacitors are suitable for dynamic loads, they have low gravimetric density of stored energy compared to batteries (Fig. 1, a compilation from [13-15]). As energy storage devices, supercapacitors have approximately ten times higher gravimetric power density but ten times lower gravimetric energy density compared to batteries. By hybridizing these into a single energy storage system, the benefits of each component can be utilized. Such systems in which batteries and supercapacitors are hybridized are called battery-supercapacitor hybrid energy storage systems (HESS) (Fig. 2. A compilation from [16-22]). From a topological perspective, the simplest form of such an energy storage system is called a passive hybrid battery supercapacitor system (a passive HBSS), or a passive hybrid battery-supercapacitor energy storage systems (a passive HESS).

The basic difference between passive HBSS and all other HBSS topologies is that all other HBSS topologies use power electronics circuits to control energy flows. The use of power electronics circuits, such as bidirectional DC/DC converters, enables better adaptation and optimization of energy flows in variable operating conditions.

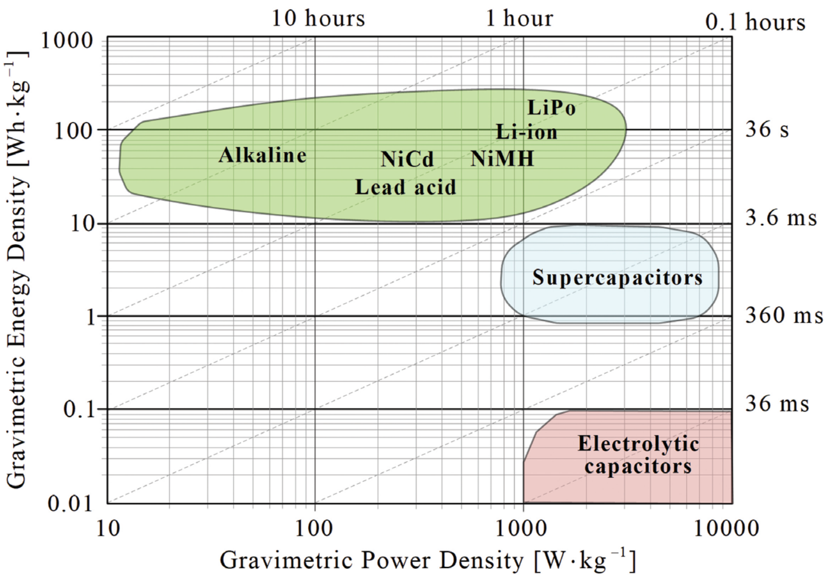

Fig. 1. Ragone plot with charge/discharge time for arbitrarily selected energy storage technologies

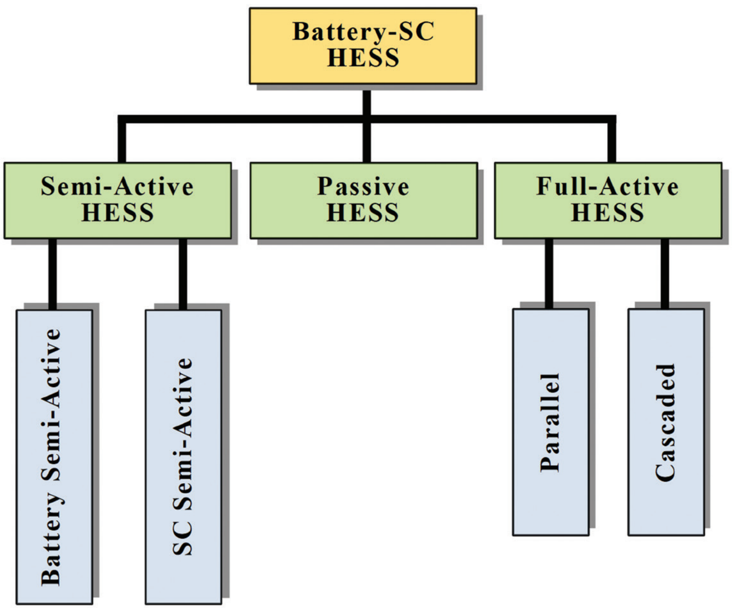

Fig. 2. Common classification of hybrid battery supercapacitor topologies

In a passive H BSS, the battery and the supercapacitor are connected in parallel, i.e. to common DC buses (Fig. 3. A compilation from [16-22]).

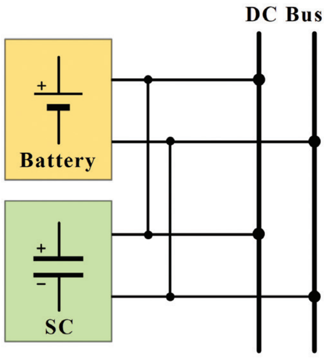

Fig. 3. Passive hybrid battery-supercapacitor topology

Compared to a stand-alone battery energy storage system, a hybrid battery supercapacitor system, or a “stand-alone battery system” (SABS), has several prominent advantages:

- Reduced battery current: In case of dynamic loads, the load current in the HBSS is distributed between HBSS components, i.e. between the battery and the supercapacitor. Therefore, the battery current inside the HBSS is reduced in comparison to the SABS [13, 23-28].
- Reduced battery temperature and thermal stress: Joule heating of the battery changes with the square of the current flowing through the battery. The released heat in the battery raises the temperature of the battery. Under dynamic load, during a significant increase in current, due to the fact that the released heat changes with the square of the current flowing through the battery, there is a sudden change in the temperature of the battery. Both a significant increase and a sudden change in battery temperature cause thermal stress on the battery. Thermal stress degrades battery performance and contributes to reduced battery lifespan. Given that the battery current inside the HBSS is reduced compared to the SABS, the temperature and thermal stress of the battery inside the HBSS are also reduced compared to the SABS [27-30].
- Increased energy efficiency and energy absorption: Regarding energy efficiency, the HBSS has higher energy efficiency under dynamic load compared to the SABS [13, 31, 32]. Furthermore, in relation to the stand-alone battery, the HBSS has an increased ability to absorb the energy delivered by the load during braking of the electric motor drive (recuperation).
- Less pronounced voltage sags: During sudden increases in load currents, the voltage at common DC buses, i.e. at the hybrid battery supercapacitor system terminals, experiences significantly less pronounced voltage sags compared to the SABS [13, 15, 28, 32].
- This paper focuses on energy efficiency, specifically analyzing the energy efficiency of the passive HBSS using analytical expressions. For this purpose, analytical expressions will be derived that enable analysis of the energy efficiency of the passive HBSS. First, the derivation of expressions for voltages and currents during dynamic loading of the passive HBSS will be presented. Using the obtained analytical expressions, the derivation of the expressions for losses in the HBSS will be presented for the scenario in which the load is dynamic. Then, the losses in the HBSS will be compared with the losses that would occur in a SABS under the same dynamic load. The energy efficiency of the passive HBSS was also analyzed for different parameters of the dynamic load. The contributions and findings of this paper can be summarized as follows:
- A novel set of analytical expressions is introduced for the analysis of the energy efficiency of the passive HBSS in relation to the SABS.
- Derived expressions are expressed through two additional parameters (K and k) that are related to the HBSS circuit parameters. This resulted in a form of equations that is suitable for rapid identification of the influence of HBSS parameters on current and voltage conditions and energy efficiency under dynamic load.
- By providing insights into the influence of HBSS parameters on current and voltage conditions and energy efficiency under dynamic load, this paper contributes to the advancement of sizing the HBSS components and optimization practices, ultimately leading to improved energy use.
- The proposed approach to analyzing the energy efficiency of the HBSS using the introduced analytical expressions resulted in the finding that there is an interval of the parameter k which is optimal for sizing the HBSS from a techno-economic aspect.

2. PASSIVE HBSS MODEL

In order for the derivation of analytical expressions to be possible, the HBSS model should be reduced to only necessary circuit elements. If the consideration is limited to very specific conditions under which the passive HBSS operates, then a straightforward model can effectively represent both the passive HBSS and the load. Such approach is often used in the performance analysis of hybrid battery supercapacitor systems [14, 22-24, 26, 27, 31-34]. We have already employed this approach in [28], and the same model will be utilized in this paper. For the purpose of presentation consistency, the main steps in modeling and derivation of analytical expressions are repeated from [28]. After repeating the steps from [28] in obtaining the analytical expressions for the HBSS currents, a derivation of analytical expressions is given for the energy efficiency specific to this paper.

Assuming that the analysis is carried out in a time interval during which the battery has not discharged significantly, the electromotive force of the battery (denoted by E), i.e. the open circuit voltage of the battery, can be considered constant.

Additionally, the battery equivalent series resistance (R_B) can be considered constant and equal for both battery charging and discharging processes. According to the above, the battery can be modeled as a real voltage source (Fig. 4 [13, 15, 22, 28]). Given that phenomena such as self-discharge and dielectric absorption are not expressed during the fast charge and discharge of a supercapacitor, the supercapacitor can be represented by a series connection of the SC capacitance (C) and the equivalent series resistance of the supercapacitor (R_C) (Fig. 4 [13, 15, 22, 28]).

According to the previously adopted models for battery and supercapacitor, the passive HBSS behaves as a first-order system during transients. Consequently, it has already been established that the transient will be described by a first-order differential equation. To further simplify the model of the entire system, it is essential to select an appropriate load model.

For this purpose, the dynamic load can be modeled with a current sink. The load current should have such a waveform that encompasses both steady and dynamic load time intervals. This can be achieved by representing the load current as a periodic pulse train that also contains a constant part (Fig. 5 [13, 15, 28]):

$$i_L = I_0 + \sum_{k=0}^{N} (I_p - I_0)\big(u(t-kT) - u(t-T_p-kT)\big), \quad (1)$$

where $u(t\text{-}kT)$ and $u(t\text{-}T_p\text{-}kT)$ are Heaviside unit step functions, I_0 is the amplitude of the constant part of the load, I_p is the amplitude of rectangular pulses, T_p is the duration of rectangular pulses, and T is the period of the waveform.

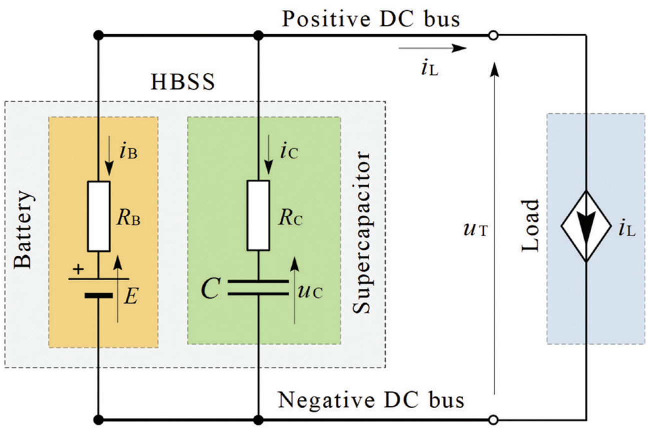

Fig. 4. Passive HBSS modeled as a first-order circuit

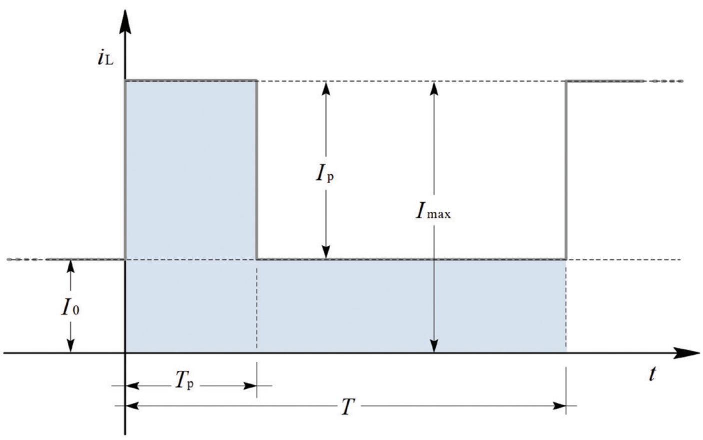

Fig. 5. Load current and relevant quantities

3. MATHEMATICAL DESCRIPTION OF TRANSIENTS

Based on the previously presented HBSS and the load model, the procedure used to obtain analytical expressions for battery and supercapacitor currents during transients will be described in this section.

According to Fig. 4, for each t there holds [28]:

$$i_L = -i_B - i_C, \quad (2)$$

$$E + i_B \cdot R_B - i_C \cdot R_C - u_C = 0, \quad (3)$$

$$i_C = C \cdot \frac{du_C}{dt}. \quad (4)$$

Combining the previous three expressions yields [28]:

$$(R_B+R_C)\cdot C\cdot\frac{du_C}{dt}+u_C=E-i_L\cdot R_B\,. \tag{5}$$

A first-order differential equation is obtained, which is usually written as follows [28]:

$$\tau\cdot\frac{du_C}{dt}+u_C=E-i_L\cdot R_B\,, \tag{6}$$

where the time constant is determined by the expression [28]:

$$\tau=(R_B+R_C)\cdot C\,. \tag{7}$$

Differential equation (6) holds for both SC transients (i.e. discharging and charging transients). The same time constant (7) applies to both transients. Depending on whether the SC transient is charging or discharging, the load current will vary in the differential equation (6).

For the SC discharge transient process ($0_+\leq t\leq Tp$), the load current is

$$i_L=I_0+I_p\,. \tag{8}$$

For the SC charging transient process ($Tp+\leq t\leq T$), the load current is

$$i_L=I_0\,. \tag{9}$$

To clearly distinguish between the voltages and currents in the HBSS associated with the charging and discharging processes of the supercapacitors, it is desirable to introduce the following notation. The voltages and currents related to the discharge process of the supercapacitor will be denoted with the index one, while those related to the charging process will be denoted with the index two.

According to the adopted notation, voltage u_C and currents i_L and i_B can be written as follows:

$$u_C=\begin{cases}u_{C1} & \text{for } 0_+\leq t\leq T_{p-},\\ u_{C2} & \text{for } T_{p+}\leq t\leq T.\end{cases} \tag{10}$$

$$i_L=\begin{cases}i_{L1}=I_0+I_p & \text{for } 0_+\leq t\leq T_{p-},\\ i_{L2}=I_0 & \text{for } T_{p+}\leq t\leq T.\end{cases} \tag{11}$$

$$i_B=\begin{cases}i_{B1} & \text{for } 0_+\leq t\leq T_{p-},\\ i_{B2} & \text{for } T_{p+}\leq t\leq T.\end{cases} \tag{12}$$

3.1. SUPERCAPACITOR DISCHARGE PROCESS

The transient phenomenon of the supercapacitor discharge begins with the occurrence of a current pulse at $t=0_+$. For the SC discharge process ($0_+\leq t\leq T_p$), the following holds:

$$\tau\cdot\frac{du_{C1}}{dt}+u_{C1}=E-(I_0+I_p)\cdot R_B\,. \tag{13}$$

Applying the Laplace transform to Eq. (13) gives

$$\tau\cdot\left[s\cdot U_{C1}(s)-u_{C1}(0_-)\right]+U_{C1}(s)=\frac{E-(I_0+I_p)\cdot R_B}{s}\,. \tag{14}$$

A minor rearrangement of the previous expression yields

$$U_{C1}(s)=\frac{u_C(0_-)}{s+\frac{1}{\tau}}+\frac{1}{T}\cdot\frac{1}{s}\cdot\frac{\left(E-(I_0+I_p)\cdot R_B\right)}{s+\frac{1}{\tau}}\,. \tag{15}$$

According to the law of commutation [35], the voltage across the capacitor remains unchanged during commutation.

$$u_C1(0_+)=u_{C1}(0_-)=E-i_L(0_-)\cdot R_B\,. \tag{16}$$

Since $i_L(0_-)=I_0$, it follows that

$$u_{C1}(0_-)=u_{C1}(0_+)=E-I_0\cdot R_B=U_0\,, \tag{17}$$

where U_0 indicates the steady-state voltage at the terminals of the HBSS. It is also convenient to define the theoretical voltage drop at the battery terminals due to the dynamic (peak) current I_p in the scenario when the SC is absent (i.e. as it would be in the SABS configuration):

$$\Delta U_p=I_p\cdot R_B\,. \tag{18}$$

Taking into account (17) and (18), expression (15) can be further simplified to the following form:

$$U_{C1}(s)=\frac{U_0}{s+\frac{1}{\tau}}+\frac{1}{\tau}\cdot\frac{1}{s}\cdot\frac{U_0-\Delta U_p}{s+\frac{1}{\tau}}\,. \tag{19}$$

Applying the inverse Laplace transform to (19) gives

$$u_{C1}=U_0-\Delta U_p\cdot\left(1-e^{-\frac{t}{\tau}}\right). \tag{20}$$

Once the expression for the voltage u_{C1} is known, it is possible to determine the current i_{C1}. Substituting expression (20) into (4) gives

$$i_{C1}=C\cdot\frac{du_{C1}}{dt}=-\Delta U_p\cdot\frac{C}{\tau}e^{-\frac{t}{\tau}}=-\frac{\Delta U_p}{R_B+R_C}e^{-\frac{t}{\tau}}\,. \tag{21}$$

Taking into account (18) yields

$$i_{C1}=-I_p\frac{R_B}{R_B+R_C}e^{-\frac{t}{\tau}}\,. \tag{22}$$

To make further expressions concise, two auxiliary parameters, k and K, can be introduced [28]:

$$k=\frac{R_B}{R_C}\,, \tag{23}$$

$$K=K(k)=\frac{R_B}{R_B+R_C}=\frac{k}{1+k}\,. \tag{24}$$

Graphical representation of expression (24), i.e. K vs. k, is shown in Figure 6 [28].

Taking into account (24), expression (22) can be written as follows:

$$i_{C1}=-I_p\cdot K\cdot e^{-\frac{t}{\tau}}\,. \tag{25}$$

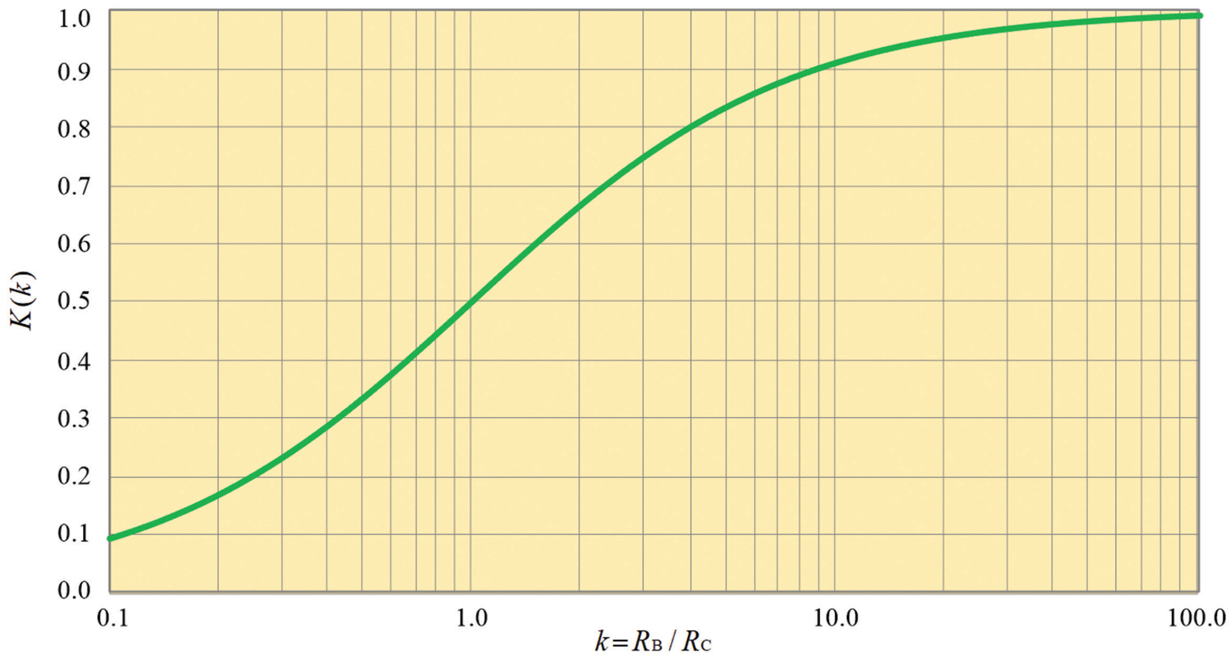

Fig. 6. Parameter K vs. parameter k

3.2. SUPERCAPACITOR CHARGING PROCESS

The transient phenomenon of supercapacitor charging begins when the current pulse ends at $t=Tp_+$. For the SC charging transient process ($T_p \leq t < T$), there holds:

$$\tau \cdot \frac{du_{C2}}{dt} + u_{C2} = E - I_0 \cdot R_B. \tag{26}$$

Taking into account that $E\text{-}I_0 \cdot R_B = U_0$, expression (26) turns into a more concise form:

$$\tau \cdot \frac{du_{C2}}{dt} + u_{C2} = U_0. \tag{27}$$

By applying the previously described procedure for solving the differential equation, the following is obtained:

$$u_{C2} = u_{C2}(T_{p-}) + \left(U_0 - u_{C2}(T_{p-})\right) \cdot \left(1 - e^{-\frac{t-T_p}{\tau}}\right). \tag{28}$$

The initial condition $u_{C2}(T_{p-})$ is determined by applying the law of commutation. According to the law of commutation [35], the voltage across the capacitor remains unchanged during commutation:

$$u_{C2}(T_{p-}) = u_{C2}(T_{p+}) = u_{C1}(T_p) = U_0 - \Delta U_p \cdot \left(1 - e^{-\frac{T_p}{\tau}}\right). \tag{29}$$

Once the expression for the voltage u_{C2} is known, it is possible to determine the current i_{C2}. Substituting expression (28) into (4) gives

$$i_{C2} = C \cdot \frac{du_{C2}}{dt} = \left(U_0 - u_{C2}(T_p)\right) \cdot \frac{C}{\tau} \cdot e^{-\frac{t-T_p}{\tau}}. \tag{30}$$

Substituting (7) into (30) gives

$$i_{C2} = \frac{U_0 - u_{C2}(T_p)}{R_B + R_C} \cdot e^{-\frac{t-T_p}{\tau}}. \tag{31}$$

As was done for the current i_{C1}, it is beneficial to describe the current i_{C2} using the parameter K. From (24), it follows that

$$\frac{1}{R_B + R_C} = \frac{K}{R_B}. \tag{32}$$

Combining (31) and (32) gives

$$i_{C2} = \frac{U_0 - u_{C2}(T_p)}{R_B + R_C} \cdot e^{-\frac{t-T_p}{T}} = \frac{K}{R_B} \cdot \left(U_0 - u_{C2}(T_p)\right) \cdot e^{-\frac{t-T_p}{T}}. \tag{33}$$

Combining (29) and (33) gives

$$i_{C2} = \frac{K}{R_B} \cdot \Delta U_p \cdot \left(1 - e^{-\frac{T_p}{\tau}}\right) \cdot e^{-\frac{t-T_p}{T}}. \tag{34}$$

By respecting the definition of ΔU_p (expression (18)), expression (34) can be written as follows:

$$i_{C2} = I_p \cdot K \cdot \left(1 - e^{-\frac{T_p}{T}}\right) \cdot e^{-\frac{t-T_p}{T}} \tag{35}$$

3.3. BATTERY CURRENT IN THE HBSS

The battery current in the HBSS during both SC transients (discharging and charging) is determined from expression (2):

$$i_B = -i_L - i_C. \tag{36}$$

Taking into account the previously introduced notation, (36) and (11) yield the following:

$$i_B = \begin{cases} -(I_0 + I_p) - i_{C1} & \text{for } 0_+ \leq t \leq T_{p-}, \\ -I_0 - i_{C2} & \text{for } T_{p+} \leq t \leq T. \end{cases} \tag{37}$$

Inserting (25) and (35) into (37) gives

$$i_{B1} = -I_0 - I_p \cdot \left(1 - K \cdot e^{-\frac{t}{\tau}}\right), \tag{38}$$

$$i_{B2} = -I_0 - I_p \cdot K \cdot \left(1 - e^{-\frac{T_p}{\tau}}\right) \cdot e^{-\frac{t-T_p}{\tau}}. \tag{39}$$

The expressions obtained for the supercapacitor and battery currents allow for an analysis of how the presence of the supercapacitor affects the redistribution of the current pulse between the supercapacitor and the battery in the passive HBSS. Specifically, this analysis focuses on the reduction of the battery's current stress in the passive HBSS. Battery current reduction and rectangular pulse current redistribution between the supercapacitor and the battery can be easily analyzed through the introduced parameter K [28]. For example, if the supercapacitor and the battery have equivalent series resistances, then k=1 follows according to (23). According to Fig. 6 for parameter k=1, the parameter K is equal to 0.5. According to expressions (25) and (38), at the moment a rectangular current pulse (t=0$_+$) appears, the pulse current I_p is evenly distributed between the supercapacitor and the battery. The supercapacitor absorbs half of the current pulse, while the battery absorbs the other half. Once all currents (i_{C1}, i_{C2}, i_{B1} and i_{B2}) are expressed through the parameter K, their waveform can be graphically represented qualitatively as a function of the parameter K (Fig. 7 [28]). The waveforms of battery and supercapacitor currents shown in Fig. 7 [28] are all consistent with the results given in the literature [14, 15, 26, 34, 36-38].

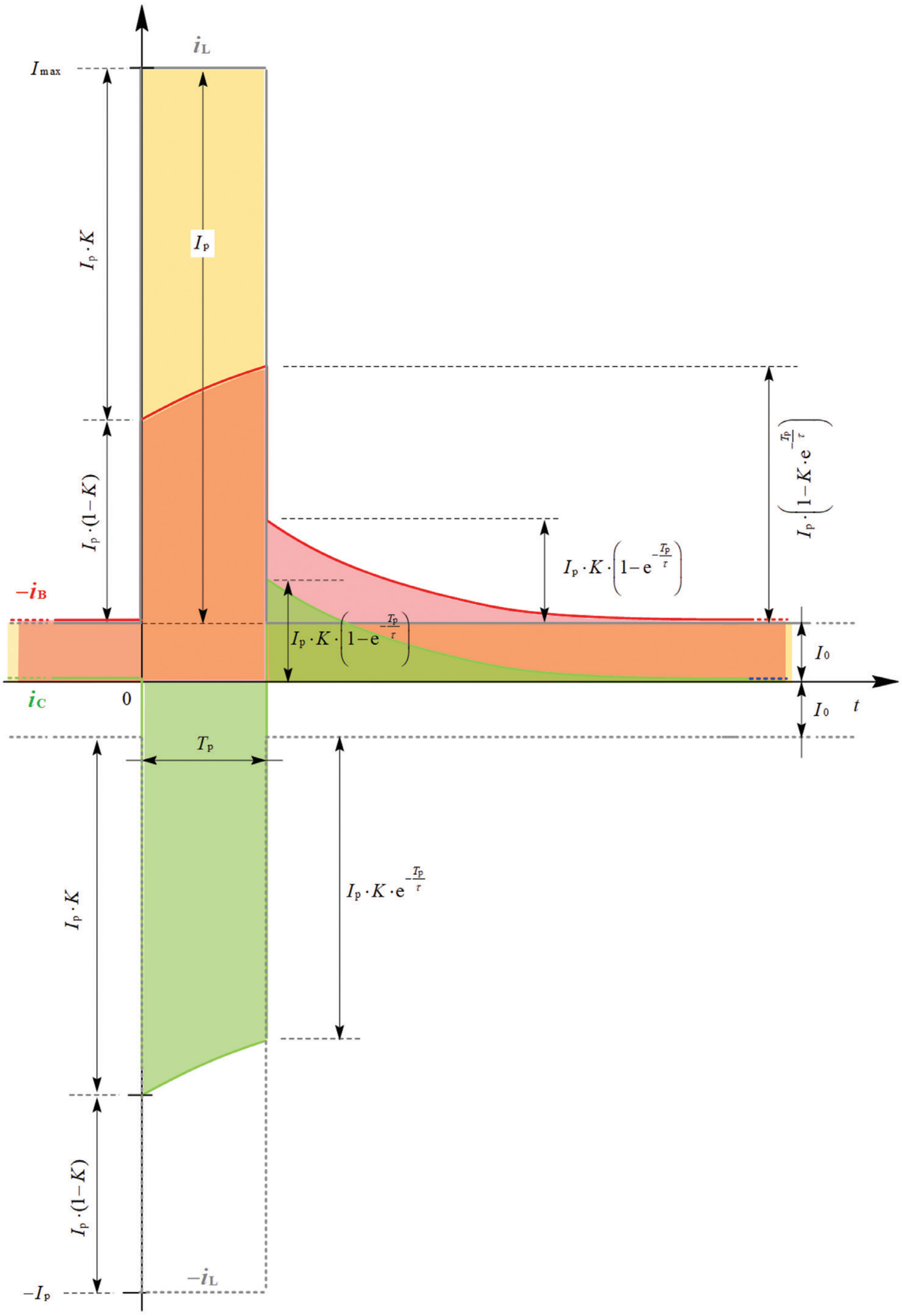

Fig. 7. Waveforms of the battery, load and SC current expressed through parameter K

If the parameter k is greater than one, then a larger portion of the rectangular pulse current is initially absorbed by the supercapacitor. More precisely, at $t=0_+$ the supercapacitor absorbs part of the pulse current, which is $Ip{\cdot}K$, and the battery absorbs the rest of the pulse current, which is $I_p{\cdot}(1\text{-}K)$ (see Fig. 7.). As long as the rectangular pulse current lasts, the current of the supercapacitor decreases, and the current of the batteries increases (see Fig. 7). As shown in Fig. 7, at the end of the rectangular pulse current, the direction of the supercapacitor current changes due to the supercapacitor charging process that occurs. The charging current of the supercapacitor is provided by the battery. Therefore, after the end of the current pulse, the battery is loaded with a current equal to the sum of the constant part of the load current I_0 and the supercapacitor charging current i_{C2}. The supercapacitor charging process takes approximately five time constants (Eq. 7). Therefore, when sizing the HBSS component, such HBSS parameters should be selected that the supercapacitor has enough time to charge before a new rectangular pulse current occurs.

4. ENERGY EFFICIENCY

To demonstrate that the HBSS is more energy efficient compared to the SABS under dynamic load, it is necessary to know the losses within both the HBSS and the SABS under the same operating conditions. The energy efficiency of the HBSS compared to a battery-only system (SABS) can be analyzed through a loss ratio:

$$\frac{E_{\mathrm{HBSS}}(0,T)}{E_{\mathrm{SABS}}(0,T)}=\frac{E_{\mathrm{HBSS}}(0,T_{\mathrm{p}})+E_{\mathrm{HBSS}}(T_{\mathrm{p}},T)}{E_{\mathrm{SABS}}(0,T_{\mathrm{p}})+E_{\mathrm{SABS}}(T_{\mathrm{p}},T)}, \tag{40}$$

where E_{HBSS} $(0,T)$ are losses in the $HBSS$ during one period of the waveform T, i.e. total losses in the battery and the supercapacitor during one period, E_{SABS} $(0,T)$ are losses in the SABS during one period T.

For the sake of mathematical simplicity and to facilitate easier interpretation of the results, it is convenient to introduce the following substitutions:

$$\varepsilon=\frac{I_{\mathrm{p}}}{I_0}, \tag{41}$$

where I_p is the peak value of the rectangular pulse and I_0 is the constant part of the load current.

The ratio of the duration of the rectangular pulse to the time constant is given by

$$\alpha=\frac{T_{\mathrm{p}}}{\tau}. \tag{42}$$

where T_p is the pulse duration and τ is the time constant.

The ratio of the duration of the rectangular pulse to the period of the waveform, the so-called duty cycle of the pulsating (dynamic) part of the load current, is given by

$$\beta=\frac{T_{\mathrm{p}}}{T}, \tag{43}$$

where T_p is the pulse duration and T is the period of the waveform.

Using the previously introduced substitutions, the battery current and the SC current can be written as

$$i_{\mathrm{B1}}=-I_0\cdot\left[1+\varepsilon\cdot\left(1-K\cdot \mathrm{e}^{-\frac{t}{\tau}}\right)\right], \tag{44}$$

$$i_{\mathrm{B2}}=I_0\cdot\left[1+\varepsilon\cdot K\cdot\left(1-\mathrm{e}^{-\frac{T_{\mathrm{p}}}{\tau}}\right)\cdot \mathrm{e}^{-\frac{t-T_{\mathrm{p}}}{\tau}}\right], \tag{45}$$

$$i_{\mathrm{C1}}=-I_0\cdot\varepsilon\cdot K\cdot \mathrm{e}^{-\frac{t}{\tau}}, \tag{46}$$

$$i_{\mathrm{C2}}=I_0\cdot\varepsilon\cdot K\cdot\left(1-\mathrm{e}^{-\frac{T_{\mathrm{p}}}{T}}\right)\cdot \mathrm{e}^{-\frac{t-T_{\mathrm{p}}}{T}}. \tag{47}$$

4.1. LOSSES IN A STAND-ALONE BATTERY SYSTEM

Losses due to the load current in a SABS over one period can be determined as follows:

$$E_{\mathrm{SABS}}(0,T)=R_{\mathrm{B}}\cdot\int_0^T i_{\mathrm{L}}^2\mathrm{d}t=R_{\mathrm{B}}\cdot I_{\mathrm{L}}^2, \tag{48}$$

where I_L^2 is given by the expression:

$$I_{\mathrm{L}}^2=\int_0^{T_{\mathrm{p}}} i_{\mathrm{L1}}^2\cdot\mathrm{d}t+\int_{T_{\mathrm{p}}}^{T} i_{\mathrm{L2}}^2\cdot\mathrm{d}t=I_{\mathrm{L1}}^2+I_{\mathrm{L2}}^2. \tag{49}$$

Integrals I_{L1}^2 and I_{L2}^2 are given by the expressions:

$$I_{\mathrm{L1}}^2=\int_0^{T_{\mathrm{p}}}(I_0+I_{\mathrm{p}})^2\cdot\mathrm{d}t=(I_0+I_{\mathrm{p}})^2\cdot\int_0^{T_{\mathrm{p}}}\mathrm{d}t, \tag{50}$$

$$I_{\mathrm{L2}}^2=\int_{T_{\mathrm{p}}}^{T} I_0^2\cdot\mathrm{d}t=I_0^2\cdot\int_{T_{\mathrm{p}}}^{T}\mathrm{d}t. \tag{51}$$

Using the previously introduced substitutions, the previous integrals can be written as follows:

$$I_{\mathrm{L1}}^2=(I_0+I_{\mathrm{p}})^2\cdot T_{\mathrm{p}}=I_0^2\cdot(1+\varepsilon)^2\cdot T_{\mathrm{p}}, \tag{52}$$

$$I_{\mathrm{L2}}^2=I_0^2\cdot(T-T_{\mathrm{p}})=I_0^2\cdot T_{\mathrm{p}}\cdot\left(\frac{T}{T_{\mathrm{p}}}-1\right)=I_0^2\cdot T_{\mathrm{p}}\cdot\left(\frac{1}{\beta}-1\right). \tag{53}$$

4.2. LOSSES IN AN HBSS SYSTEM

Losses in the HBSS due to the load current consist of losses in the battery and the supercapacitor. The total energy loss in the HBSS from the moment $t=0$ to the moment T is given by

$$E_{\mathrm{HBSS}}(0,T)=E_{\mathrm{B}}(0,T)+E_{\mathrm{SC}}(0,T), \tag{54}$$

where E_B $(0,T)$ and E_{SC} $(0,T)$ denote the losses in the battery during one period and the losses in the supercapacitor during one period, respectively.

The losses in the battery during one period can be determined as

$$E_{\mathrm{B}}(0,T)=E_{\mathrm{B}}(0,T_{\mathrm{p}})+E_{\mathrm{B}}(T_{\mathrm{p}},T), \tag{55}$$

where

$$E_B(0,T_p) = R_B \cdot \int_0^{T_p} i_{B1}^2 \mathrm{d}t = R_B \cdot I_{B1}^2 \,, \quad (57)$$

The losses in the supercapacitor during one period can be determined as

$$E_{SC}(0,T) = E_{SC}(0,T_p) + E_{SC}(T_p,T) \,, \quad (58)$$

where

$$E_{SC}(0,T_p) = R_C \cdot \int_0^{T_p} i_{C1}^2 \mathrm{d}t = R_C \cdot I_{C1}^2 \,, \quad (59)$$

$$E_{SC}(T_p,T) = R_C \cdot \int_{T_p}^{T} i_{C2}^2 \mathrm{d}t = R_C \cdot I_{C2}^2 \,, \quad (60)$$

where I_{B1}^2, I_{B2}^2, I_{C1}^2 and I_{C2}^2 denote the following integrals:

$$I_{B1}^2 = I_0^2 \cdot T_p \cdot \left[\begin{aligned} &(1+\varepsilon)^2 - 2\cdot\varepsilon\cdot(1+\varepsilon)\cdot K \cdot \left(\frac{1}{\alpha}\right)\cdot\left(1-e^{-\alpha}\right) + \\ &+\varepsilon^2 \cdot K^2 \cdot \left(\frac{1}{2}\frac{1}{\alpha}\right)\cdot\left(1-e^{-2\alpha}\right) \end{aligned} \right], \quad (61)$$

$$I_{B2}^2 = I_0^2 \cdot T_p \cdot \left[\begin{aligned} &\left(\frac{1}{\beta}-1\right) + 2\varepsilon K \alpha \cdot \left(1-e^{-\alpha}\right)\cdot\left(1-e^{-\frac{1-\beta}{\beta}\alpha}\right) + \\ &+(\varepsilon\cdot K)^2 \cdot \left(\frac{\alpha}{2}\right)\cdot\left(1-e^{-\alpha}\right)^2 \cdot \left(1-e^{-2\frac{1-\beta}{\beta}\alpha}\right) \end{aligned} \right], \quad (62)$$

$$I_{C1}^2 = I_0^2 \cdot T_p \cdot (\varepsilon \cdot K)^2 \cdot \left(\frac{1}{2}\frac{1}{\alpha}\right)\cdot\left(1-e^{-2\alpha}\right), \quad (63)$$

$$I_{C2}^2 = I_0^2 \cdot T_p \cdot (\varepsilon \cdot K)^2 \cdot \left(1-e^{-\alpha}\right)^2 \cdot \left(\frac{1}{2}\frac{1}{\alpha}\right)\cdot\left(1-e^{-2\alpha\frac{1-\beta}{\beta}}\right). \quad (64)$$

Up to this point, all losses have been expressed by the corresponding integrals. By inserting the corresponding integrals into (40), the expression for the energy efficiency can be expressed in the form:

$$\frac{E_{HBSS}(0,T)}{E_{SABS}(0,T)} = \frac{R_B \cdot I_{B1}^2 + R_B \cdot I_{B2}^2 + R_C \cdot I_{C1}^2 + R_C \cdot I_{C2}^2}{R_B \cdot I_{L1}^2 + R_B \cdot I_{L2}^2}. \quad (65)$$

According to (23), i.e. $k=R_B/R_C$, expression (65) can be simplified and written as

$$\frac{E_{HBSS}(0,T)}{E_{SABS}(0,T)} = \frac{I_{B1}^2 + I_{B2}^2 + \frac{1}{k}\left(I_{C1}^2 + I_{C2}^2\right)}{I_{L1}^2 + I_{L2}^2}. \quad (66)$$

The previous expression allows for the determination of the energy efficiency of the HBSS in relation to the SABS. For practical application of this expression, it is necessary to adhere to the initial assumption under which it was derived. To ensure its applicability, it is necessary to determine the validity limits of the expression for energy efficiency.

4.3. VALIDITY LIMITS OF THE EXPRESSION FOR ENERGY EFFICIENCY

The initial assumption before deriving the expressions was that the duration of the rectangular current pulse T_p is relatively short compared to the period T. This means that after the rectangular current pulse ends, the supercapacitor manages to recharge during the time interval T-T_p.

According to the above, and given that the time required for the supercapacitor to recharge to 99% of the voltage on its terminals in a steady state is equivalent to five time constants, the following condition follows:

$$T - T_p \geq 5 \cdot \tau \,, \quad (67)$$

or expressed through the introduced substitutions:

$$\alpha \geq 5 \cdot \frac{\beta}{1-\beta}. \quad (68)$$

The previous expression enables the calculation of the range of valid parameters for determining energy efficiency (Table 1).

Table 1. Valid parameter ranges

$\beta \leq 1/10$	0.01	0.02	0.03	0.04	0.05
$\alpha \geq 5(\beta/(1-\beta))$	0.051	0.102	0.155	0.208	0.263

Table 1. Valid parameter ranges - Continued

$\beta \leq 1/10$	0.06	0.07	0.08	0.09	0.1
$\alpha \geq 5(\beta/(1-\beta))$	0.319	0.376	0.435	0.495	0.556

By adhering to the limits specified in Table 1, within which the expression for determining the energy efficiency of the HBSS in relation to the SABS holds, a numerical calculation was carried out (Figs. 8 and 9).

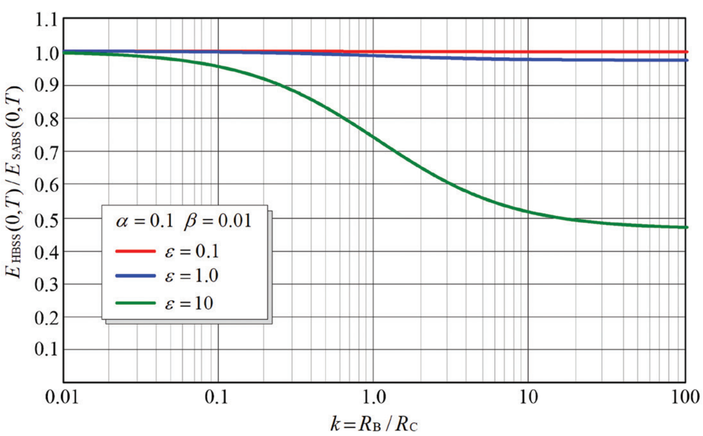

Fig. 8. Reduction of energy losses in the HBSS compared to a stand-alone battery system for α=0.1, β=0.01 and a variation of ε.

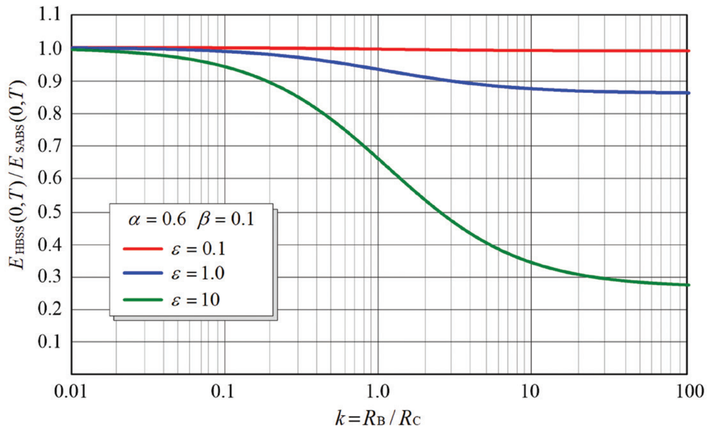

Fig. 9. Reduction of energy losses in the HBSS compared to a stand-alone battery system for α=0.6, β=0.1 and a variation of ε.

5. DISCUSION

In Section 3, analytical expressions are given that describe the battery and SC currents in the passive HBSS. The presented analytical expressions enable a physical explanation of the reasons why the HBSS is more energy efficient than the SABS under dynamic load. At the moment a rectangular current pulse appears, i.e. the dynamic component of the load current (see Fig. 7), the pulse current is distributed between the battery and the supercapacitor. Given that supercapacitors typically have a lower internal resistance than a battery (k>1), a larger part of the pulse current is absorbed by the SC (see Fig. 7). The current delivered by the SC to the load during the current pulse generates losses in the SC; however, since the internal resistance of the SC is lower than that of the battery, the losses in the HBSS are also reduced compared to the SABS.

For a more detailed analysis of the energy efficiency of the HBSS relative to the SABS, it is necessary to use the expressions provided in Section 4. This section includes analytical expressions that account for all losses in the HBSS. The expressions enable the numerical calculation of the energy efficiency of the HBSS relative to the SABS for various dynamic load parameters.

The numerical calculation of the energy efficiency of the HBSS relative to the SABS was performed over a range of parameter k from 0.01 to 100. Although the range of parameter k from 0.01 to 100 has no practical importance, as it would include extremes (a large battery and a small SC, a small battery and a large SC), the specified range was chosen for purely theoretical reasons. The selected range of parameter k enables recognition of the asymptotic values towards which the curves in figures converge.

The red curves in Figs. 8 and 9 refer to cases when there is a very small dynamic current component in the total load current, i.e. when $\varepsilon=I_p/I_0=0.1$. As expected, the ratio $E_{HBSS}(0,T)/E_{SABS}(0,T)$ is approximately one, i.e. the losses in the HBSS are not reduced compared to the SABS. The HBSS is as energy efficient as the SABS.

With an increase in the share of the dynamic current component (blue curves in Figs. 8 and 9), i.e. when $\varepsilon=I_p/I_0=1.0$, the advantages of the HBSS compared to the SABS become obvious, i.e. the losses in the HBSS compared to the SABS are smaller (a smaller $E_{HBSS}(0,T)/E_{SABS}(0,T)$ ratio). By comparing Figs. 8 and 9, it can be observed that the reduced losses in the HBSS compared to the SABS are not significant if the pulse duration is small compared to the period ($\beta=T_p/T=0.01$ in Fig. 8).

By increasing the duration of the pulse T_p compared to the period T (a greater $\beta=T_p/T$ ratio) and a simultaneous increase in the share of the dynamic current component in the load current (a greater $\varepsilon=I_p/I_0$ ratio), the advantages of the HBSS in terms of energy efficiency become obvious (the green curve in Fig. 9).

Furthermore, following the green curves in Fig. 8 and Fig. 9, asymptotic trends can be observed with the change of the parameter k. The physical interpretation is that there is a physical limit to how much the losses in the HBSS can be reduced, and consequently, how much the energy efficiency of the HBSS can be improved. For practical cases where the supercapacitor has a lower internal resistance than the battery (k>1), up to approximately $k\approx10$, the green curve shows significant changes with variations in the parameter k up to approximately $k\approx10$. Beyond this point, i.e. for k>10, the green curves exhibit pronounced asymptotic behavior, the so-called saturation. This indicates that there is an interval of the parameter k, i.e. $1\leq k\leq10$, which is optimal for the sizing of the HBSS from a techno-economic aspect. Reducing losses in the HBSS, or increasing the energy efficiency of the HBSS, by choosing a parameter k greater than 10 becomes uneconomical. This would lead to an HBSS design with a relatively large supercapacitor compared to the battery. In addition, the reduction of losses would be small compared to the reduction of losses with the range of k values between 1 and 10, i.e. for $1\leq k\leq10$.

The results obtained are consistent with those reported in the literature [13, 27]. However, the approach proposed in this paper offers a more straightforward interpretation of the results and provides better insights into how HBSS parameters affect current and voltage conditions and energy efficiency under dynamic load. The conducted analysis points to the following conclusions.

If the dynamic component of the load current is a very small part of the total load current, or if load currents vary slowly, the passive HBSS does not offer any advantages over the SABS in terms of energy efficiency. The reduction in battery current within the HBSS compared to the SABS is negligible. Because of this, there are no techno-economic justifications for using the HBSS in these cases. The analysis also showed that the HBSS is more advantageous than the SABS in applications where the dynamic current component constitutes a significant portion in the total load current (a greater I_p/I_0 ratio). Additionally, from a techno-economic aspect, there is an interval of the parameter , i.e. ($1 \le k \le 10$), which is optimal for the sizing of the HBSS. Besides improving the energy efficiency compared to the SABS, the HBSS also offers other advantages compared to the SABS, including a lower voltage drop at the HBSS terminals, and reduced current and thermal stress on the battery inside the HBSS.

6. CONCLUSIONS

This paper presents an analytical approach to the analysis of energy efficiency of the passive HBSS in relation to the SABS. A novel set of analytical expressions is introduced for this analysis. Analytical expressions are derived using a first-order circuit model that represents both the HBSS and the dynamic load. These derived expressions are expressed through two additional auxiliary dimensionless parameters (K and k) that are related to the circuit parameters of the HBSS. The waveform of the load current is characterized by three dimensionless parameters (ε, α, and β), one of which (α) is also related to the HBSS parameters. Thanks to the above, general expressions for currents, voltages and energy efficiency have been achieved. Compared to existing approaches, the proposed approach offers better insight into the influence of battery and supercapacitor parameters on the energy efficiency of the passive HBSS in relation to the SABS. The proposed approach is also suitable for quick determination of HBSS behavior (voltage at the HBSS terminals, battery and SC currents) and energy efficiency under dynamic load. This enables the identification of HBSS parameters required for optimal behavior and optimal energy efficiency. In this way, it is possible to avoid successive simulations with varying model parameters to determine the behavior and energy efficiency of the HBSS under dynamic load. Instead, if necessary, more accurate results can be obtained through simulations based on a higher-order model.

The performed analysis illustrates the effectiveness of the proposed approach in determining the energy efficiency of the HBSS in relation to the SABS, as well as the rapid identification of the HBSS parameters required for desired behavior and energy efficiency. Regarding energy efficiency, the analysis reveals that there is an interval of the parameter k which is optimal for the sizing of the HBSS a techno-economic aspect. The advantages of the passive HBSS compared to the SABS are described in detail in the paper. The analysis shows that the passive HBSS has advantages over the SABS only under dynamic load. More specifically, the passive HBSS is more suitable than the SABS in applications where the dynamic current component constitutes a significant portion in the total load current (a greater I_p/I_0 ratio).

The presented set of analytical expressions can be further improved for greater accuracy by using a more complex circuit model to represent the HBSS. To pave the way for potential improvement, the derivation of this set of analytical expressions is provided in detail, accompanied by explanatory comments.

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