Enhanced MO-PSO Preview Control of a Non-Linear Inverted Pendulum System

Original scientific paper

Nidhika Birla
Department of Electrical Engineering
National Institute of Technology
Krurkshetra, Haryana, India
e-mail: nidhikabirla@gmail.com

Akhilesh Swarup
Department of Electrical Engineering
National Institute of Technology
Krurkshetra, Haryana, India
e-mail: akhilesh.swarup@gmail.com

Abstract *– The use of advanced knowledge of reference can improve the tracking quality and performance of transient response of a control system and the systems using such information are called preview control systems. This paper explains the implementation of preview control methodology available in literature, i.e. DARE and LMI-based approaches for a non-linear system. The design of a multi-objective preview controller for a non-linear inverted pendulum system based on the MO-PSO (Multi-Objective Particle Swarm Optimization) algorithm. The proposed algorithm uses a modified objective function to deal with multiple objectives and replaces infeasible solutions by feasible ones with the help of the concept of brood generation in the MBO (Marriage in Honey Bees) algorithm. The use of the notion of brood generation strengthens the algorithm by making the solutions not fall into local optima. The design methods are verified, for time response and robustness and stability parameters, on a benchmark nonlinear model of inverted pendulum. Simulations are carried out on a MATLAB platform. The test results verify the potential of the proposed MO-PSO procedure as per the quality of the solution obtained in terms of multiple objectives considered for the design problems.*

Keywords *– discrete algebraic Riccati equation (DARE), linear matrix inequality (LMI), marriage in honey bees optimization (MBO), particle swarm optimization (PSO), preview control.*

1. INTRODUCTION

Preview control is a field well suited for application to systems that have reference signals known a priori. The use of advance knowledge of a reference signal can improve the tracking quality of the concerned control system. The field of preview control has attracted many researchers as its applications include guidance of autonomous vehicles, robotics and process control. A full information linear longitudinal missile guidance problem is posed and solved by Farooq and Limbeer [1] using a linear quadratic regulator (LQR) framework and state-space augmentation techniques. This work is extended to the output feedback case by Farooq et al. [2]. Marzbanrrad et al. [3] considered the design of an active earthquake suspension system for a building, under the assumption that remote measurements of ground vibrations are available. Automatic path planning and handling dynamic environments are the key areas of research in the field of robotics. The use of future information on the path to be tracked (curvature of the path) is proposed to be used for improvement in performance of the tracking system for wheeled mobile robots by Kadakkal et al. [4]. The use of preview control is also reported for the precise control of various process elements, some of them being a linear brushless DC motor [5], an electromechanical valve actuator [6] and a three-phase induction motor [7]. Despite a large number of publications on the subjects of preview control, no discussions show the capability of use of preview control for non-linear systems. This paper presents the use of classical preview control theory for non-linear systems. It also presents a novel PSO-based approach to design of a multi-objective preview controller for a non-linear inverted pendulum system.

A classical solution of the preview control problem is given by using H_∞ control and state augmentation, solved using the algebraic Riccati equation. The mathematical formulation and solution of the H_∞ preview control problem is given by A. Kojima et al. and G. Tadmor et al., for preview compensation, output feedback setting and fixed lag smoothing [8-11]. A discrete version of the preview control problem and its various issues are studied with numerical examples by Polyakov et al. [12]. Y. Kuroiwa et al. have analysed the H_∞ preview control problem for the systems with delay [13]. Analysis and design of H_∞ preview tracking control systems and their various variations using state augmentation have also been studied [1, 14-15]. The solutions to all problems are given using the algebraic Riccati equation for the continuous and discrete algebraic Riccati equation for discrete – time systems. Takaba [16-17] addressed the problem of the robust LQ/H_∞ servomechanism design with preview action for polytopic uncertain systems using linear matrix inequalities. Liao [18] has also designed the LMI-based preview control against actuator faults.

This paper discusses the implementation of preview control methodology available in literature, i.e. DARE and LMI-based approaches for a non-linear system. The design of a multi-objective preview controller for a non-linear system based on the MO-PSO (Multi-Objective Particle Swarm Optimization) technique is also presented and analysed in this paper. The paper is organised in five sections. The first section introduces literature and the objective of the paper. The solution methodologies for the preview control problem available in literature and the non-linear system model is explained in the second section. The next section introduces the multi-objective preview controller design methodology based on the improved PSO technique. The results are summed up in the fourth section and the conclusion is drawn in the last section.

2. PRELIMINARIES

A simple SISO preview tracking problem is shown in Figure 1. The preview control system has its control signal dependent on the present error between the reference signal and system output and the future information available for the reference signal or the disturbance and the present condition of system states.

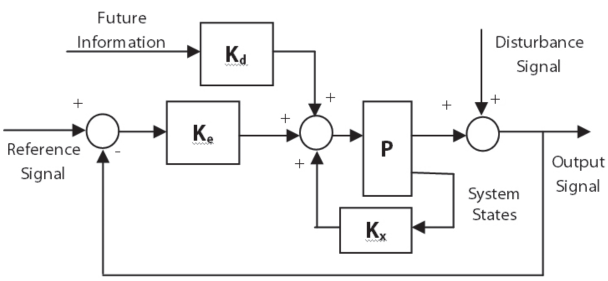

Fig. 1. Structure of the SISO Preview Tracking System

A general discrete-time system is described by

$$x(t+1) = Ax(t) + Bu(t) + Ed(t) \tag{1-a}$$

$$z(t) = Cx(t) + Du(t), \tag{1-b}$$

where $x(t) \epsilon R^n$ and $u(t) \epsilon R^m$ are the state vector and control input, respectively. The signal $z(t) \epsilon R^p$ denotes the controlled output or the tracking error. Moreover, $d(t) \epsilon R^l$ denotes the exogenous signal which can be considered as the reference signal or disturbance.

The purpose of preview control is to design a static controller in the form of

$$u(t) = K_x x(t) + K_e e(t) + \sum_{i=0}^{h} K_{di} d(t+i), \tag{2}$$

so that the following quadratic performance index is made satisfactorily small even in the presence of the exogenous input d.

$$J = \| z \|_2^2 = \sum_{i=0}^{\infty} \| z(t) \|^2 \tag{3}$$

The first and second term on the right-hand side of controller equation (2) represent the static state feedback and preview compensation, respectively.

2.1. SOLUTION METHODOLOGIES

The solution of the above stated preview control problem can be obtained by state augmentation. The previewed information and the error signal between the reference and present output signal is made part of the system states. The augmented system now becomes

$$\xi(t+1) = F\xi(t) + Gu + Ld(t), \tag{4-a}$$

$$z(t) = H\xi(t) + Du(t), \tag{4-b}$$

and the augmented state vector is defined as

$$\xi(t) = \left[x^T(t)\ \ x_e^T(t)\ \ x_d^T(t)\right]^T. \tag{5}$$

The standard robust controller theory can now be invoked to design a preview controller for the augmented system.

2.1.1. METHOD I: USING DARE

The discrete algebraic ricatti equation is one of the solutions to such problem and is explained by Takaba in his work [15]. For a positive constant γ, there exists a stabilizing state feedback control with preview action satisfying the H∞ performance level

$$\sup \frac{\| z \|_2}{\| d \|_2} < \gamma \tag{6}$$

if and only if there exists a positive semi-definite stabilizing solution P to the following algebraic Riccati equation such that $W := \gamma^2 - L^T P L > 0$.

$$P = F^T PF - \begin{bmatrix} G^T PF + D^T H \\ L^T PF \end{bmatrix}^T$$
$$v^{-1} \begin{bmatrix} G^T PF + D^T H \\ L^T PF \end{bmatrix} + H^T H \tag{7-a}$$

$$V = \begin{bmatrix} D^T D & 0 \\ 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} G^T \\ L^T \end{bmatrix} P \begin{bmatrix} G & L \end{bmatrix} \tag{7-b}$$

In this case, one of the desired H_∞ preview controllers is given by

$$\begin{bmatrix} K_x & K_{do} & \ldots & K_{dh} \end{bmatrix}$$
$$= -(D^T D + G^T \hat{P} G)^{-1} (G^T \hat{P} F + D^T H) \tag{8-a}$$

$$\hat{P} = P + PLW^{-1} L^T P \tag{8-b}$$

2.1.2. METHOD II: USING LMIS

The LMI theory can also be applied to the augmented system for optimizing the solution for either H_2 or H_∞ or both norms of the system [16-18]. The augmented system (4-a & 4-b) modified by applying state feedback control $u(t)=K\xi(t)$ that yields

$$\xi(t+1) = F_K \xi(t) + Lw(t), \tag{9-a}$$

$$\zeta(t) = H_k \xi(t), \tag{9-b}$$

where $F_K=F+GK$ and $H_K=H+JK$.

For a mixed H_2-H_∞ problem, it is assumed that the solution matrices of H_2 and H_∞ LMIs are the same, i.e. $X=X_2=X_\infty$. This results in a conservative solution as a solution matrix has to satisfy both LMIs for the H_2 and H_∞ norm simultaneously. The improvement in the result is possible if the above assumption is not made. The problem statement is described as below

$$min(sup(w_1 \| T_2 \|^2 + w_2 \| T_\infty \|^2)), \tag{10}$$

subject to the constraints

$$F_K^T X F_K - X + L^T L < 0 \tag{11}$$

$$\begin{bmatrix} F_K & L \\ H_K & 0 \end{bmatrix}^T \begin{bmatrix} X & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} F_K & L \\ H_K & 0 \end{bmatrix} - \begin{bmatrix} X & 0 \\ 0 & -\gamma^2 I \end{bmatrix} < 0, \tag{12}$$

$$X > 0 \tag{13}$$

where $w1$ and $w2$ are the weights assigned by the designer, $||T_2||$ is the highest limit on the H_2 norm of the system (represented by η) and $||T_\infty||$ is the highest limit on the H_∞ norm of the system (represented by γ).

2.2. SYSTEM FORMULATION

The inverted pendulum system is a complex multivariable, non-minimum and unstable, electromechanical system with severe non-linearity and its stabilizing control is the representative problem in the application of the control theory. It can reflect many significant control problems, such as the study of stability, non-linear system control and the robustness problem. Because of its characteristics, the inverted pendulum system is used to verify the efficacy of the control theory. This paper studies the effectiveness of preview control for the inverted pendulum system.

The inverted pendulum system [19] is depicted in Figure 2 and it consists of an inverted pendulum mounted on a motor-driven cart. The inverted pendulum is unstable in that it may fall over any time in any direction unless a suitable control force is applied. An assumption is made that the pendulum mass is concentrated at the end of the rod (the rod is massless) and the control force is applied to the cart. The system has one input, i.e. control force u, and two outputs, namely, pendulum angle θ and cart position x.

The goal for the system is to keep the pendulum upright as much as possible and yet control the position of the cart.

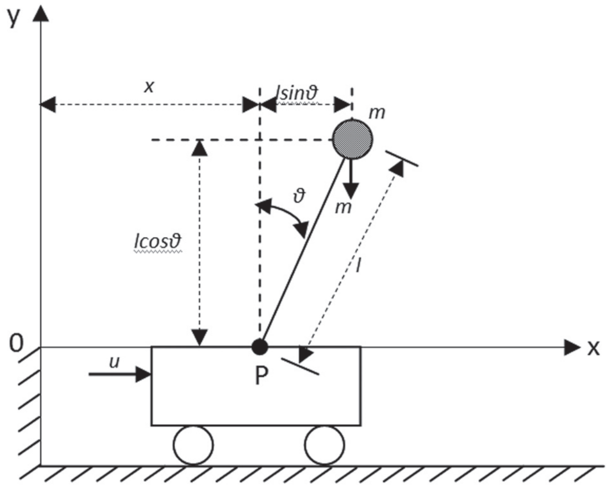

Fig. 2. Inverted Pendulum System

With reference to Figure 2, applying Newton's second law to the system motion results in the following non-linear dynamics of the system:

$$(M+m)\ddot{x} - ml(\sin\dot{\Theta}^2 + ml(\cos\Theta)\ddot{\Theta} = u \tag{14-a}$$

$$m\ddot{x}\cos\Theta + ml\ddot{\Theta} = mg\sin\Theta \tag{14-b}$$

Non-linear equations, with linearization condition $\Theta \approx 0$, can be linearized as

$$M\ddot{x} + mg\Theta \backsimeq u \tag{15-a}$$

$$Ml\ddot{\Theta} - (M + m) g\Theta \approx - u \qquad (15\text{-b})$$

State variables of the system are defined by, $x_1=\Theta$, $x_2=\dot{\Theta}$, $x_3=x$, $x_4=\dot{x}$, where angle Θ indicates the rotation of the pendulum about point P and x is the location of the cart. From these definitions of variables and above equations the state space system model can be framed. The discrete state space model of the system described by above equations is obtained with a sampling interval of $ts=0,1$ seconds. The preview controllers are designed using the linearized and discretized model of the inverted pendulum but are verified for effectiveness on its nonlinear model.

3. IMPROVED MO-PSO PREVIEW CONTROL DESIGN

A study about the criterion considered for preview controller design reveals that the considered transient behaviour is a secondary feature, as the objective function does not include the term that is directly a function of transient behaviour in the optimization problem using DARE and LMI based solutions. The classical solution methodology is not flexible enough to simultaneously include different types of objective functions like minimize cost, maximize performance, maximize reliability, etc. Thus, in this section, a PSO based multi-objective optimization algorithm is presented for preview controller design of the non-linear (discretized and linearized) inverted pendulum system.

Particle swarm optimization (PSO), a heuristic technique for automated optimization of multi-dimensional problems, was introduced by Kennedy and Eberhart [25-26]. It is inspired by the birds' flocking behaviour. Instead of using evolutionary operators to manipulate the individuals, each individual in PSO flies in the search space with a velocity which is dynamically adjusted according to its own flying experience and its companions' flying experience. Each individual is treated as a volume-less particle (a point) in the D-dimensional search space. The ith particle is represented as $X_i=(x_{i1}, x_{i2},...,x_{iD})$. The best previous position (the position giving the best fitness value) of the ith particle is recorded and represented as $P_i=(p_{i1},p_{i2},...,p_{iD})$. The index of the best particle among all the particles in the population is represented by the symbol g. The rate of position change (velocity) for particle i is represented as $V_i=(v_{i1}, v_{i2},...v_{iD})$. The particles are manipulated according to the following equation:

$$v_{id}^{(t+1)} = w \bullet v_{id}^{(t)} + c_1 \bullet rand() \bullet (pbest - x_{id}^{(t)})$$
$$+ c_2 \bullet rand() \bullet (gbest - x_{id}^{(t)}) \qquad (16\text{-a})$$

The variable w is the inertia weight, c_1 and c_2 are positive constants and $rand()$ is a random function. Equation (16-a) is used to calculate the new velocity according to its previous velocity and distance of its current position from both its best historical position and its neighbours' best position. Then, the particle flies towards the new position according to equation (16-b). The process is repeated until a user-defined stopping criterion is reached.

3. 1. FITNESS EVALUATION

The PSO procedure requires a definition of objectives that specify the design requirements. The objectives considered in this paper are the H_∞ norm of the system (to guarantee robustness) $J_1=\|G(z)\|_\infty$ the H_2 norm of the system (to guarantee noise insensitivity behaviour) $J_2=\|G(z)\|_2$ and the Integral of Absolute Error (IAE) (to tackle transient behaviour) $J_3=\int|G(z)|$. As the numerical values of these objectives will be in different ranges, so the values of these objectives are normalized with respect to the maximum value of the respective objective function that provides the fitness of the solution on a normalized range. The normalized objective functions are then combined using the weighted sum approach to deal with multiple-objectives and the objective function is represented as

$$\min(z) = w_1 z'_1(x) + w_2 z'_2(x) + ... + w_n z'_n(x) \qquad (17)$$

where $z'_1(x)$ is a normalized objective function. The weights w_i are assigned values depending on the design preferences, for equal preference all the weights are kept as 1. The advantage of this approach is its easy implementation and computational efficiency that allows the optimization algorithm to search for a globally optimal solution.

3.2. IMPROVED MO-PSO ALGORITHM

The design method using a standard PSO algorithm is enhanced with an observation that it is difficult to tune the controller for a nonlinear system with multiple objectives. The standard PSO algorithm is most likely to fall in local optima for multi-objective problems. So, the concept of generation of broods in the MBO (Marriage in Honey Bees Optimization [27]) algorithm is applied to obtain new solutions to replace the infeasible solutions and make the PSO find the global optimum solution. The proposed procedure starts with the random initialization of the preview controller gains that are treated as particles in the population. The design procedure for the synthesis of the preview controller using the multi-objective PSO algorithm, with modification of infeasible solutions using the MBO technique of generating broods, is presented as follows:

i. Initialize a population (array) of particles with random position and velocities on a d-dimension in problem space.

ii. For each particle, evaluate the designed optimization fitness function (as per description in equation 17) in d variables.

iii. Compare particle's fitness evaluation with particle's *pbest*. If the current value is better than pbest, then set *pbest* value equal to the current value and the *pbest* location equal to the current location in $d-$dimensional space.

iv. Compare fitness evaluation with the population's overall previous *gbest*. If the current value is better than *gbest*, then reset *gbest* to the current particle's array index and value.

v. Change the velocity and position of the particle according to equations shown in (16).

vi. If the new position particle (i.e. new solution) obtained results in a stable system, then jump to step (*x*), else replace the particle with a new generated particle, as explained in steps (*vii*) to (*ix*).

vii. Create a set of sub-optimal solutions obtained in the optimization process, i.e. a set of global best and local best values of the particles, and treat it as a set of queens.

viii. Generate a random solution in the solution space and treat it as a drone.

ix. Randomly select a queen from the set for mating. Then, probabilistically use crossover and mutation operators to generate broods. From the broods generated, randomly select a brood that gives a feasible solution to replace the infeasible solution.

x. Loop to step (*ii*) until a criterion is met, usually a sufficiently good fitness or a maximum number of iterations (generations).

For a complete system design using the proposed procedure, the fitness evaluation of each particle is carried out by using simulations on the non-linear system model for the practical reference inputs, i.e. step and sine signals that present the tracking results completely. The pseudo-codes for MO-PSO, PSO and MBO algorithms are given in Appendix A, B and C, respectively.

4. RESULTS AND DISCUSSIONS

The benchmark control system inverted pendulum is solved for an optimal preview controller using the proposed procedure on the MATLAB platform with the help of the YALMIP toolbox for the LMI design procedure. The results are summed up as follows:

From the results of repeated simulations with change in preview length '*h*', it was found that the optimal preview length for the inverted pendulum system is 5 steps. An increase in the preview length beyond 5 steps does not produce any appreciable change in the objective function, so the results are presented for $h=5$. The optimal values of error weight and control weight are chosen from repeated simulations as $Q=1$and $R=0.1$. Due to space constraints, the results of these simulations could not be presented. The proposed procedure was executed with the reference signal,

$$r(t) = \begin{cases} 1, (t \geq 50) \\ 0, (t < 50) \end{cases} \tag{18}$$

The solution obtained using DARE could not provide a stabilizing result, as shown in Figure 3.

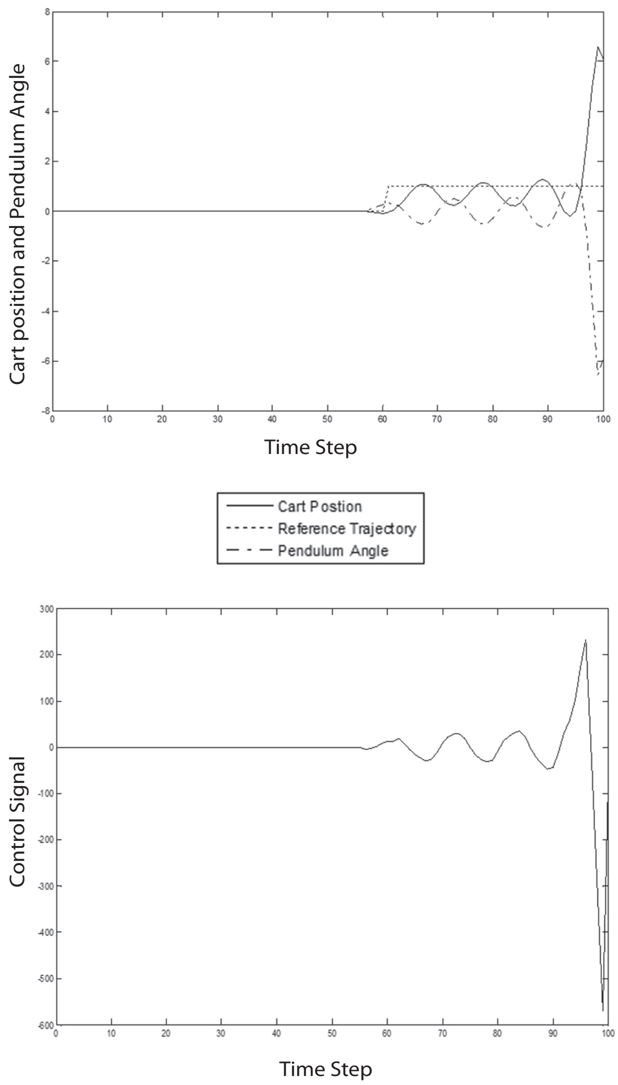

Fig. 3. Transient Response and Control Signal Variation of Inverted Pendulum with a Preview Controller obtained using DARE

The results for the preview controller obtained using LMI for mixed H_2 and H_∞ norm optimization are as shown in Figure 4.

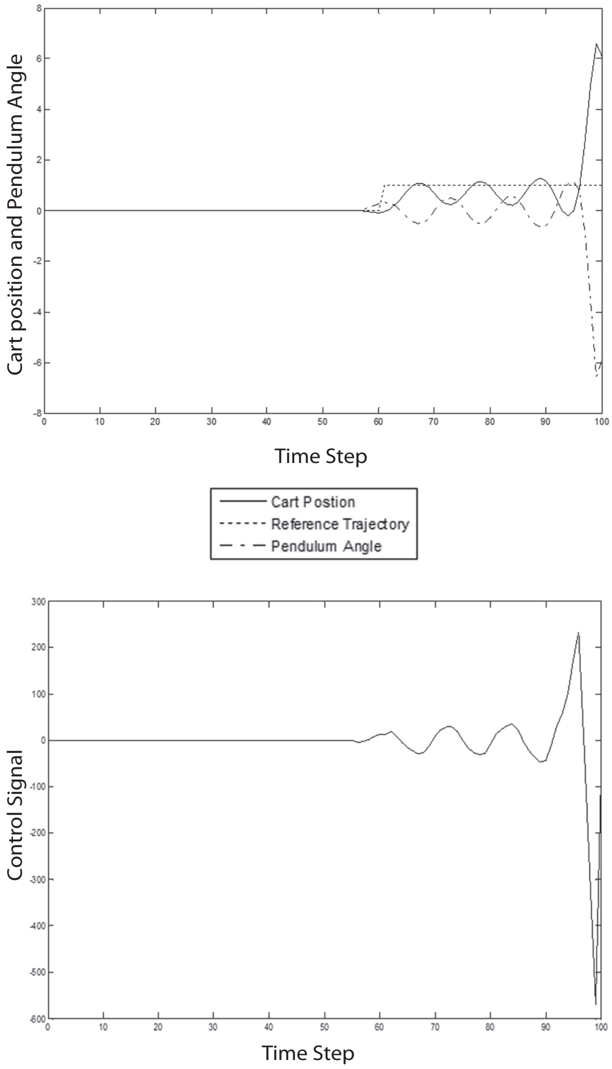

Fig. 4. Transient Response and Control Signal Variation of Inverted Pendulum with a Preview Controller obtained using LMI for Multiple Objectives

The response of the inverted pendulum for the preview controller obtained using the proposed procedure for multiple objectives (H_2, H_∞ and IAE) is shown in Figure 5. The fitness evaluation of the results is carried out based on the objective function as given in equation 17. Equal weightage is given to all objectives, so the weights of each of them are 1. The maximum value of each objective is assumed to be 10, as per the acceptable design requirements.

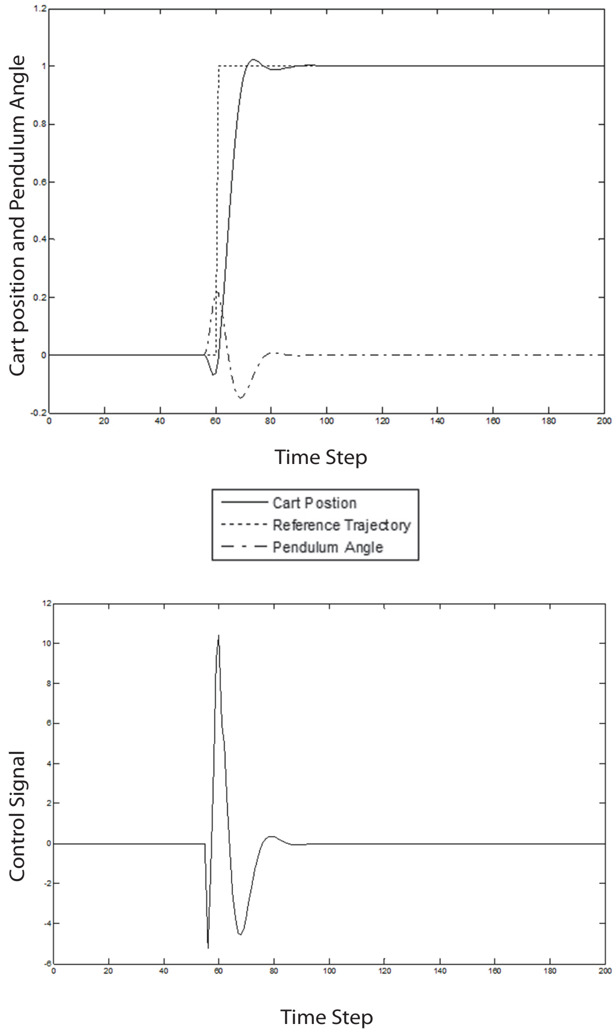

Fig. 5. Transient Response and Control Signal Variation of Inverted Pendulum with a Preview Controller obtained using the Proposed MO-PSO Procedure for Multiple Objectives

The objective function values and the quality of results in terms of robustness (γ), noise insensitivity (η) and transient behaviour (IAE) are summed up in Table 1, as given below.

The solutions obtained for LMI and the proposed MO-PSO procedure were also tested for the reference signal,

$$r(t) = \begin{cases} \sin(t), (t \geq 50) \\ 0, (t < 50) \end{cases} \tag{16-a}$$

Table 1. Comparison of System Characteristics designed by using different methods

Objective	Solution obtained using DARE	Solution obtained using LMI	Solution obtained using the proposed MO-PSO procedure
h	-	1.5442	4.1739
c	-	5.3542	6.3025
IAE	∞	62.6991	5.2758
Objective function (as per equation 17)	∞	6.9598	1.5752

and the results are shown in Figures 6 and 7.

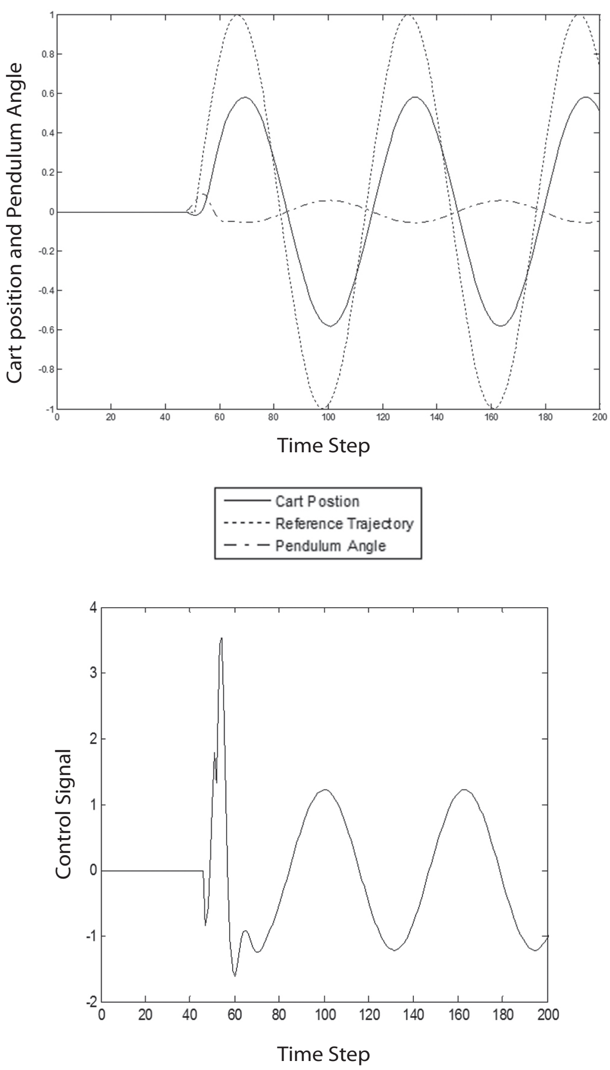

Fig. 6. Transient Response and Control Signal Variation of Inverted Pendulum with a Preview Controller obtained using LMI for Multiple Objectives

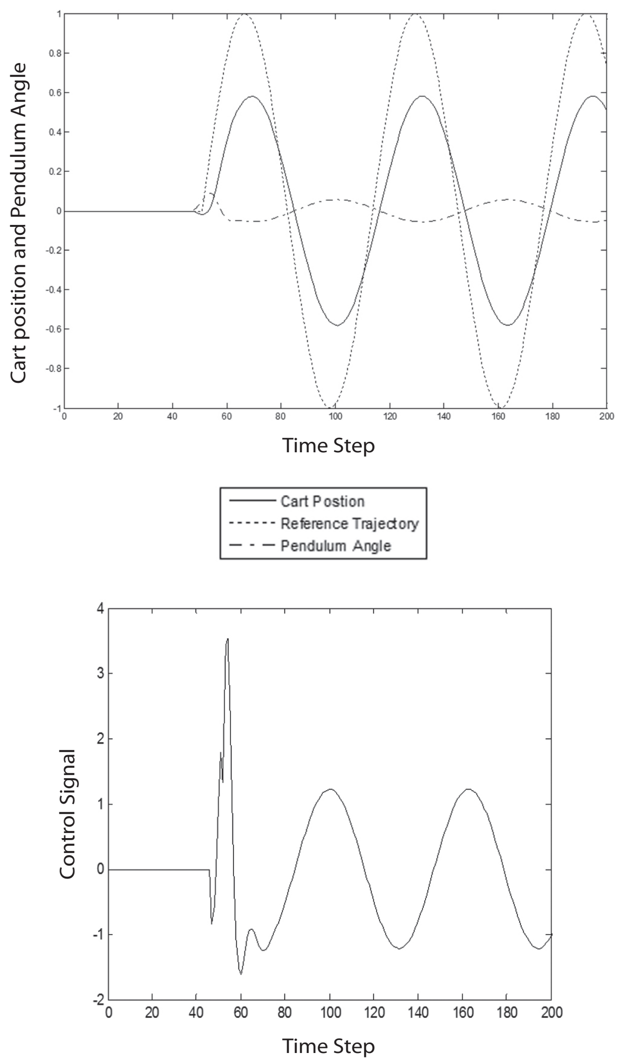

Fig.7. Transient Response and Control Signal Variation of Inverted Pendulum with a Preview Controller obtained using the Proposed MO-PSO Procedure for Multiple Objectives

The results obtained show that the proposed MO-PSO procedure successfully designs the preview controller with multiple objectives for the non-linear inverted pendulum system. The quality of solution in terms of robustness, noise insensitivity and transient behavior obtained using this procedure is remarkably better than other procedures on multi-objective front. It also takes less effort with the controller designed using the proposed procedure in comparison to other procedures. The solution obtained using DARE is strongly dependant on the γ value specified while the LMI based solution optimizes the γ and η values included in the objective function. The computation time for LMIs increases with the increase in the number of objectives as the number of LMIs increases. Also, the result found is conservative as a necessary assumption is made that the solution matrix of all LMIs is equal. The proposed MO-PSO procedure uses user-defined acceptable limits of the objective function and finds the solution that optimizes the values of all objective functions. The solution time of this procedure remains almost the same with the increase in the number of objective functions. The results obtained using this procedure are not conservative because no constraint is assumed for the gain matrices as in the LMI procedure. Also, the simulation based fitness evaluation does not require the system model to be linearized before the controller design.

5. CONCLUSION

This paper presents the design and verification of classical preview control theory for a non-linear inverted pendulum system. Also, it presents a novel MO-PSO algorithm to design preview control with multiple objectives. The procedure presents a fresh technique, to incorporate the solution replacement for infeasible solutions and also to make the particles not to fall into local optima, using the brood generation method of the MBO algorithm. The proposed MO-PSO procedure has a constant computation time even with the increase in the number of objectives as compared to the LMI procedure. The results of the procedure fly in the complete solution space as no assumption is made for a conservative result as in the LMI procedure. This method optimizes the value of the objective function as per the acceptable limit of the user. The solution obtained using the proposed procedure is better than the DARE based and LMI based methodologies to design a preview controller on multi-objective front. The quality of the solution is judged in terms the H2 norm, the H∞ norm and IAE values that present robustness, noise insensitivity and transient response characteristics of the designed system. Inclusion of multiple constraints for real-time non-linear systems is the task under progress.

6. REFERENCES

[1] A. Farooq, D. J. N. Limebeer, "Path Following of Optimal Trajectories using Preview Control", Proceedings of the 44th IEEE Conference on Decision and Control and the European Control Conference, Seville, Spain, 12 – 15 December 2005, pp. 2787-2792, 2005.

[2] A. Farooq, D.J.N. Limebeer, P.J. MacBean. "Optimal Trajectory Tracking using an Output Feedback Preview Controller", Proceedings of the IEE Seminar on Target Tracking: Algorithms and Applications, Birmingham, UK, 7 – 8 March 2006, pp. 91-98, 2006.

[3] J. Marzbanrad, G. Ahmadi, H. Zohoor, Y. Hojjat, "Stochastic Optimal Preview Control of a Vehicle Suspension", Journal of Sound and Vibration, Vol. 275, No. 3-5, pp. 973–990, 2004.

[4] V. Kadakkal, G. Cook, "Use of Preview Control Scheme with Knowledge of Future Trajectory Information for a Lane Tracking Controller on a Wheeled Mobile Robot", Proceedings of the 34th Anuual Conference of IEEE on Industrial Electronics Society, Orlando, FL, USA, 10 – 13 November 2008, pp. 1692-1697, 2008.

[5] T. Egami, T. Tsuchiya, "Disturbance Suppression Control with Preview Action of Linear DC Brushless Motor", IEEE Transactions on Industrial Electronics, Vol. 2, No. 5, pp. 494-500, 1995.

[6] L. Mianzo, H. Peng, "Output Feedback H∞ Preview Control of an Electromechanical Valve Actuator", IEEE Transactions on Control Systems Technology, Vol. 15, No. 3, pp. 428-437, 2007.

[7] M.M. Negm, J.M. Bakhashwain, M.H. Shwehdi, "Speed Control of a Three-Phase Induction Motor based on Robust Optimal Preview Control Theory", IEEE Transactions on Energy Conservation, Vol. 21, No. 1, pp. 77-84, 2006.

[8] Kojima, S. Ishijima, "H∞ Control with Preview Compensation", Proceedings of the 38th American Control Conference, Albuquerque, NM, USA, 4 – 6 June 1997, pp. 1692-1697, 1997.

[9] Kojima, S. Ishijima, "H∞ Preview Tracking in Output Feedback Setting", Proceedings of the 38th Conference on Decision and Control, Pheonix, AZ, USA, 7 – 10 December 1999, pp. 3162-3164, 1999.

[10] Tadmor, L. Mirkinn, "H∞ Preview Control and Fixed- Lag Smoothing I: Matrix ARE Solutions in Continuous-Time Systems", Proceedings of the 42nd IEEE Conference on Decision and Control, Maui, HI, USA, 9 – 12 December 2003, pp. 6515-6520, 2003.

[11] Tadmor, L. Mirkinn, "H∞ Preview Control and Fixed- Lag Smoothing II: Matrix ARE Solutions in Discrete-Time Systems", Proceedings of the 42nd IEEE Conference on Decision and Control, Maui, HI, USA, 9 – 12 December 2003, pp. 6521-6526, 2003.

[12] K. Y. Polyakov, E. N. Rosenwasser, B. P. Lampe, "Optimal Open-Loop Tracking using Sampled Data System with Preview", Proceedings of the 11th IEEE Mediterranean Conference on Control and Automation, Rhode, Greece, 18 – 20 June 2003, CD: IV04-03, 2003.

[13] Y. Kuroiwa, H. Kimura, "H∞ Control with Preview and Delay", Proceedings of the 2004 American Control Conference, Boston, MA, USA, 30 June – 2 July 2004, pp. 4794-4799, 2004.

[14] A. Moran, Y. Mikami, M. Hayase, "Analysis and Design of H∞ Preview Tracking Control Systems", Proceedings of the 4th International Workshop on Advanced Motion Control, Tsu-City, Japan, 18 – 21 March 1996, Vol. 2, pp. 482-487, 1996.

[15] K. Takaba, "A Tutorial on Preview Control Systems", SICE 2003 Annual Conference, Fukui, Japan, 4 – 6 August 2003, Vol. 2, pp. 1388-1393, 2003.

[16] K. Takaba, "Robust Preview Tracking Control for Polytopic Uncertain Systems", Proceedings of the

37th IEEE Conference on Decision and Control, Tampa, FL, USA, 16 – 18 December 1998, pp. 1765-1770, 1998.

[17] K. Takaba, “Robust servomechanism with preview action for polytopic uncertain systems”, International Journal of Robust Nonlinear Control, Vol. 10, No. 2, pp. 101-111, 2000.

[18] F. Liao, J. L. Wan, G. H. Yang, “LMI-Based Reliable Robust Preview Tracking Control Against Actuator Faults”, Proceedings of the 2001 American Control Conference, Arlington, VA, USA, 25 – 27 June 2001, pp. 1047-1052, 2001.

[19] M. Gopal, “Digital Control and State Variable Methods”, Tata-McGraw Hill, New Delhi, Second Reprint, 1997.

[20] J. Hauser, A. Saccon, R. Frezza, “On the Driven Inverted Pendulum”, Proceedings of the 44th Conference on Decision and Control and the European Control Conference 2005, Seville, Spain, 12 – 15 December 2005, pp. 6176-6180, 2005.

[21] J. C. Geromel, R.H. Korogui, J. Bernussou, “H2 and H∞ Robust Output Feedback Control for Continuous Time Polytopic Systems”, IET Control Theory and Apllications, Vol. 1, No. 5, pp. 1541-1549, 2007.

[22] G.A. Medrano-Cedra, “Robust Computer Control of an Inverted Pendulum”, IEEE Control Systems Magazine, Vol. 19, No. 3, pp. 58-67, 1999.

[23] W. Jun-Feng, L. Chun-tao, “Robust Output Feedback Control of Inverted Pendulum”, Proceedings of the 2nd IEEE Conference on Industrial Electronics and Applications, Harbin, China, 23 – 25 May 2007, pp. 1027-1030, 2007.

[24] M.I.H. Nour, J. Ooi, K.Y. Chan, “Fuzzy Logic Control vs. Conventional PID Control of an Inverted Pendulum Robot”, International Conference on Intelligent & Advanced Systems, Kuala Lumpur, Malaysia, 25 – 28 November 2007, pp. 209-214, 2007.

[25] J. Kennedy, R. Eberhart, “Particle Swarm Optimization”, Proceedings of the 1995 IEEE International Conference on Neural Networks, Perth, Australia, 27 November – 1 December 1995, Vol. IV, pp. 1942-1948, 1995.

[26] Y. Shi, R. C. Eberhart, “Empirical Study of Particle Swarm Optimization”, Proceedings of the 1999 Congress on Evolutionary Computation (CEC 99), Washington DC, USA, 6 – 9 July 1999, Vol. 3, pp. 1945-1950, 1999.

[27] H.A. Abbass, “MBO: Marriage in Honey Bees Optimization A Haplometrosis Polygynous Swarming Approach”, Proceedings of the 2001 Congress on Evolutionary Computation 2001 (CEC 2001), Seoul, Korea, 27 – 30 May 2001, Vol. 1, pp. 207-214, 2001.